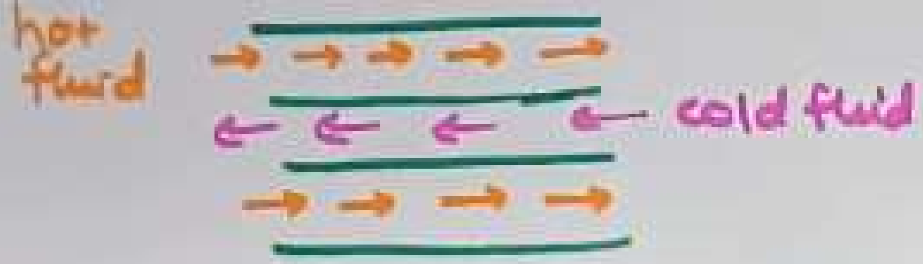
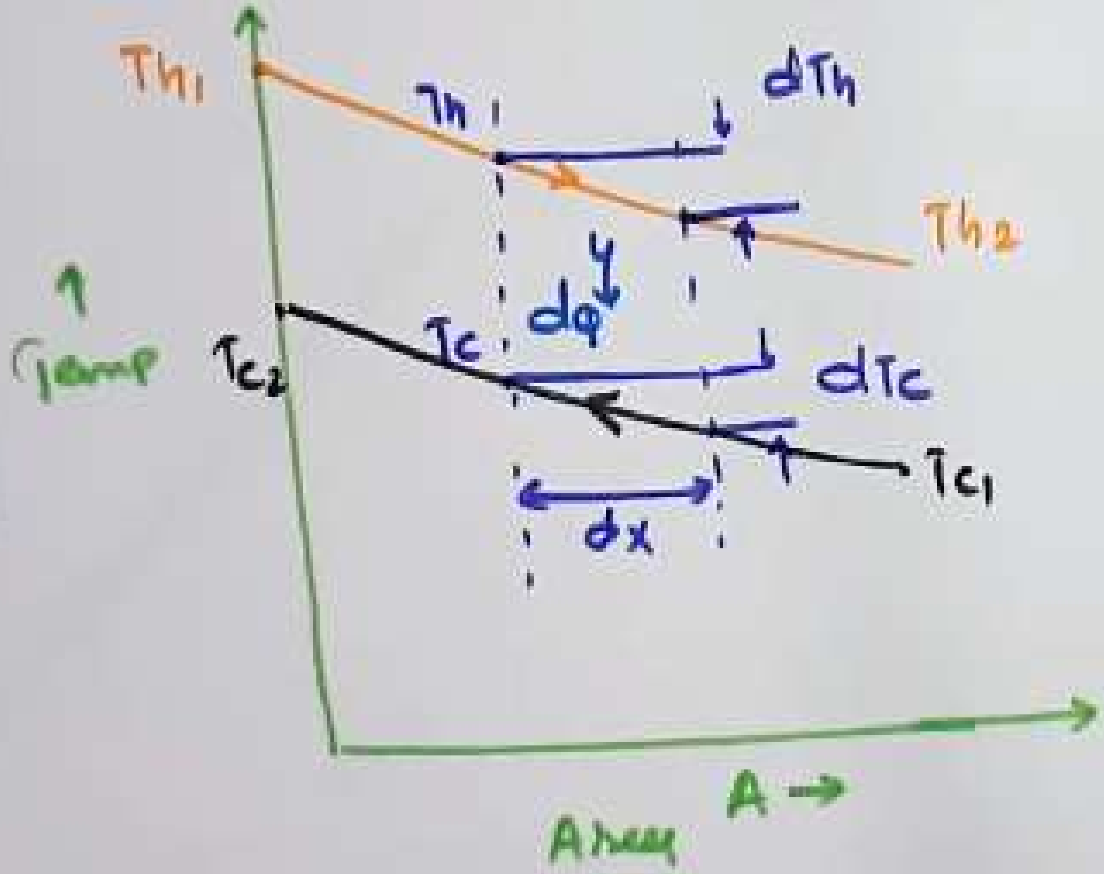
LMTD for Counter flow Heat Exchanger



flow Arrangement



$$dq = U\,dA\,(T_h - T_c)$$

$$= U\,dA\,\theta$$

$$dq = -m_h c_h\, dT_h$$

$$= -m_c c_c\, dT_c$$

$$dT_h = -\frac{dq}{m_h c_h}$$

$$dT_c = -\frac{dq}{m_c c_c}$$

$$dT_h - dT_c = -\frac{dq}{m_h c_h} - \left(-\frac{dq}{m_c c_c}\right)$$

$$d\theta = -dq\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$d\theta = -d\varphi \left[\frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$d\varphi = U\,dA\,\theta$$

Put value of $d\varphi$

$$d\theta = -U\,dA\,\theta \left[\frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$\frac{d\theta}{\theta} = -U\,dA \left[\frac{1}{C_h} - \frac{1}{C_c} \right]$$

integrating above eq^n from $A=0$ to $A=A$

$$\int_1^2 \frac{d\theta}{\theta} = \left[\frac{1}{C_h} - \frac{1}{C_c} \right] \int_{A=0}^{A=A} -U\,dA$$

$$\ln[\theta]_1^2 = \left[\frac{1}{C_h} - \frac{1}{C_c} \right] -UA$$

$$\ln \theta_2/\theta_1 = -UA \left[\frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$\varphi = m_h c_h (T_{h1} - T_{h2})$$

$$= m_c c_c (T_{c2} - T_{c1})$$

$$= C_h (T_{h1} - T_{h2})$$

$$= C_c (T_{c2} - T_{c1})$$

$$\frac{1}{C_h} = \frac{T_{h1} - T_{h2}}{\varphi}$$

$$\frac{1}{C_c} = \frac{T_{c2} - T_{c1}}{\varphi}$$

$$\ln \frac{\theta_2}{\theta_1} = -UA \left[\frac{T_{h1} - T_{h2}}{\varphi} - \frac{T_{c2} - T_{c1}}{\varphi} \right]$$

$$\ln \frac{\theta_2}{\theta_1} = -UA\left[\frac{T_{h_2} - T_{h_1}}{Q} - \frac{T_{c_2} - T_{c_1}}{Q}\right]$$

$$\ln \frac{\theta_2}{\theta_1} = -\frac{UA}{Q}\left[(T_{h_1} - T_{c_2}) - (T_{h_2} - T_{c_1})\right]$$

$$= -\frac{UA}{Q}(\theta_1 - \theta_2)$$

$$= \frac{UA}{Q}(\theta_2 - \theta_1)$$

$$Q = UA \frac{\theta_2 - \theta_1}{\ln \theta_2/\theta_1}$$

$$Q = UA\,\theta_m$$

$$\theta_m = LMTD = \frac{\theta_2 - \theta_1}{\ln \theta_2/\theta_1} = \frac{\theta_1 - \theta_2}{\ln \theta_1/\theta_2}$$