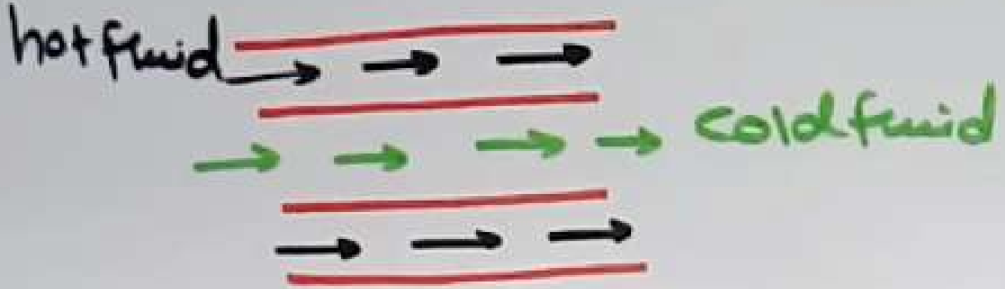

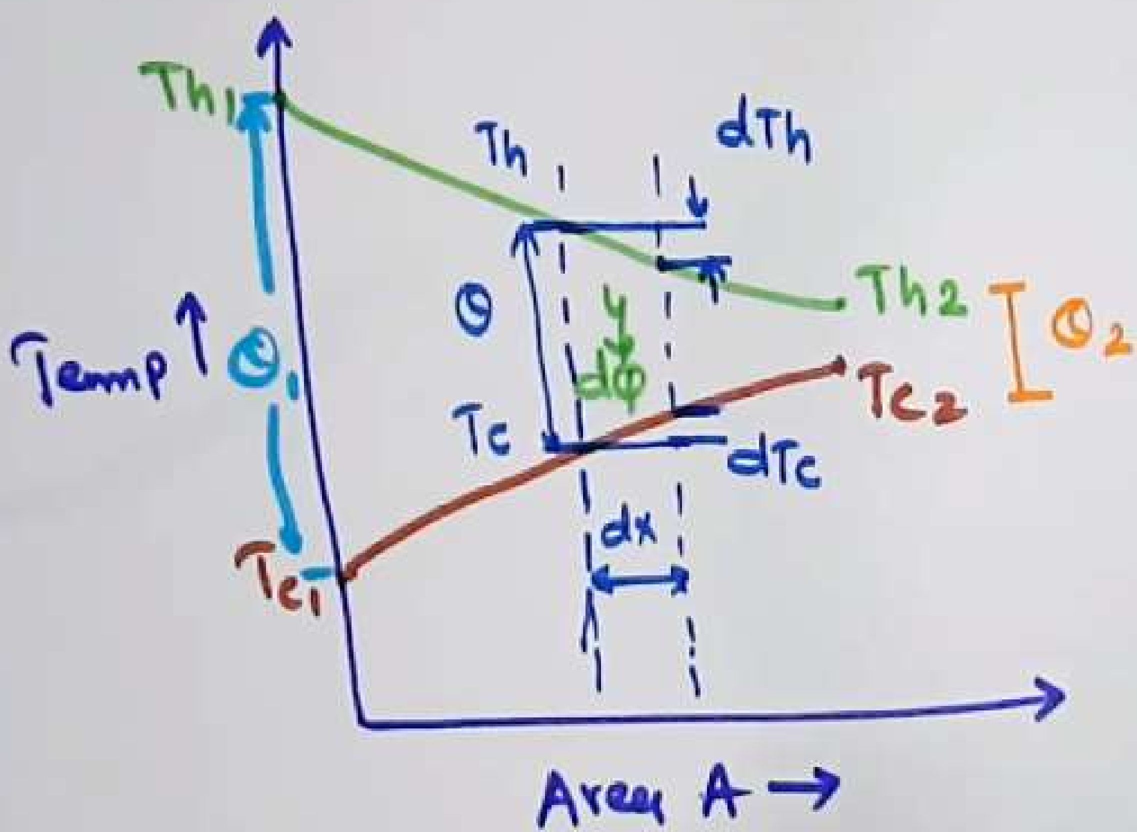

$$dq = U\,dA\,(T_h - T_c)$$

Assumption

1) U overall Heat Transfer coefficient is constant
2) Steady flow
3) Heat Exchanger is perfectly insulated.
4) No conduction along the length of Heat Exchanger
5) Kinetic and Potential energy change are neglected.

$$d\phi = U\,dA\,(T_h - T_c)$$
$$= U\,dA\,\theta$$

$$d\phi = -m_h c_h\, dT_h \quad (-ve)$$
$$= m_c c_c\, dT_c \quad (+ve)$$

$$dT_h = \frac{-d\phi}{m_h c_h} = \frac{-d\phi}{C_h}$$

$$dT_c = \frac{d\phi}{m_c c_c} = \frac{d\phi}{C_c}$$

where
$C_c = m_c c_c$ = heat capacity of cold fluid
$C_h = m_h c_h$ = heat capacity of hot fluid

$$dT_h - dT_c = \frac{-d\phi}{C_h} - \frac{d\phi}{C_c}$$

$$dT_h - dT_c = -d\phi\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d\theta = -d\phi\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d\theta = -U\,dA\,\theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d\theta}{\theta} = -U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

integrating both inlet and outlet

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\int_{A=0}^{A=A} U\,dA$$

$$\left[\ln\theta\right]_1^2 = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln[\theta_2/\theta_1] = -UA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

Total heat Transfer rate betn Two fluid

$$\phi = \dot{m}_h c_h (T_{h1}-T_{h2})$$
$$= m_c c_c (T_{c2}-T_{c1})$$

$$\phi = C_h (T_{h1}-T_{h2})$$
$$= C_c (T_{c2}-T_{c1})$$

$$\frac{1}{C_h} = \frac{T_{h1}-T_{h2}}{\phi}$$

$$\frac{1}{C_c} = \frac{T_{c2}-T_{c1}}{\phi}$$

Substituting The value of $\frac{1}{C_h}$ & $\frac{1}{C_c}$

$$\ln \theta_2/\theta_1 = -UA\left[\frac{T_{h1}-T_{h2}}{\phi}+\frac{T_{c2}-T_{c1}}{\phi}\right]$$

$$\ln \frac{\theta_2}{\theta_1} = \frac{UA}{\phi}\left[(T_{h2}-T_{c2})-(T_{h1}-T_{c1})\right]$$

$$\ln \frac{\theta_2}{\theta_1} = \frac{UA}{\phi}(\theta_2-\theta_1)$$

$$\phi = \frac{UA(\theta_2-\theta_1)}{\ln \theta_2/\theta_1}$$

$$\phi = UA\,\theta_m$$

$$\Delta T_m = \theta_m = \frac{\theta_2-\theta_1}{\ln \theta_2/\theta_1}$$