Q.1 June 2017

Using Buckingham's π- theorem show that the velocity through a circular orifice is given by,

$$V = \sqrt{2gH}\ \phi[\frac{D}{H}, \frac{\mu}{\rho V H}]$$

Where H is head causing flow, D is the diameter of the orifice, μ is the coefficient of viscosity, ρ is mass density and g is the acceleration due to gravity.

⇒ Velocity V is depends on H, D, μ, ρ, g

$$V = f(H, D, \mu, \rho, g)$$

$$f_1(V, H, D, \mu, \rho, g) = 0$$

Total no. of variable $n = 6$

No. of Dimension $m = 3$

$n - m = 6 - 3 = 3 \rightarrow \pi$ - Term

$m = 3 \Rightarrow$ no. of Repeating variable

$m + 1 = 4 \Rightarrow$ total of variable in π-Term

V	m/s	$M^0 L^1 T^{-1}$
H	m	$M^0 L^1 T^0$
D	m	$M^0 L^1 T^0$
μ	$M \cdot s/m^2$	$M^1 L^{-1} T^{-1}$
ρ	kg/m^3	$M^1 L^{-3} T^0$
g	m/s^2	$M^0 L^1 T^{-2}$

$$\pi_1 = H_1^{a_1} g_1^{b_1} \rho^{c_1} V$$

$$\pi_2 = H_1^{a_2} g_1^{b_2} \rho^{c_2} D$$

$$\pi_3 = H_1^{a_3} g_1^{b_3} \rho^{c_3} \mu$$

1) geometric Property H
2) flow Property g
3) fluid Property ρ

$$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1} V$$

$$M^0 L^0 T^0 = (L^1)^{a_1} (L^1 T^{-2})^{b_1} (M^1 L^{-3})^{c_1} (L^1 T^{-1})$$

$$= L^{a_1} L^{b_1} T^{-2b_1} M^{c_1} L^{-3c_1} L^1 T^{-1}$$

$$M^0 L^0 T^0 = M^{c_1} L^{a_1 + b_1 - 3c_1 + 1} T^{-2b_1 - 1}$$

$c_1 = 0$ $\qquad$ $c_1 = 0$

$0 = a_1 + b_1 - 3c_1 + 1 \Rightarrow 0 = a_1 - 1/2 - 0 + 1 \Rightarrow a_1 = -1/2$

$0 = -2b_1 - 1$ $\qquad$ $b_1 = -1/2$

$$\pi_1 = H^{-1/2} g^{-1/2} \rho^0 V \Rightarrow \boxed{\pi_1 = \frac{V}{\sqrt{gH}}}$$

$$\Pi_2 = H^{a_2} g^{b_2} \rho^{c_2} D$$

$$M^0 L^0 T^0 = (L^1)^{a_2} (L^1 T^{-1})^{b_2} (M^1 L^{-3})^{c_2} (L^1)$$

$$= L^{a_2} L^{b_2} T^{-b_2} M^{c_2} L^{-3c_2} L^1$$

$$M^0 L^0 T^0 = M^{c_2} L^{a_2+b_2-3c_2+1} T^{-b_2}$$

$$\Pi_2 = H^{-1} g^0 \rho^0 D$$

$$\Pi_2 = \frac{D}{H}$$

$c_2 = 0$

$0 = a_2 + b_2 - 3c_2 + 1 \Rightarrow a_2 = -1$

$b_2 = 0$

$$\Pi_3 = H^{a_3} g^{b_3} \rho^{c_3} \mu$$

$$M^0 L^0 T^0 = (L^1)^{a_3} (L^1 T^{-1})^{b_3} (M^1 L^{-3})^{c_3} (M L^{-1} T^{-1})$$

$a_3 = -3/2$

$b_3 = -1/2$

$c_3 = -1$

$$\Pi_3 = H^{-3/2} g^{-1/2} \rho^{-1} \mu = \frac{\mu}{H^{3/2} \rho \sqrt{g}}$$

$$f_1(\Pi_1 \Pi_2 \Pi_3) = 0$$

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \frac{\mu}{H H^{1/2} \rho \sqrt{g}} \right) = 0$$

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \frac{\mu}{H \rho V} \cdot \frac{V}{\sqrt{gH}} \right) = 0$$

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \frac{\mu}{V \rho H} \Pi_1 \right) = 0$$

$$\frac{V}{\sqrt{gH}} = \phi\left(\frac{D}{H}, \frac{\mu}{V \rho H} \Pi_1 \right)$$

$$\boxed{V = \sqrt{gH}\, \phi\left(D/H, \mu/V\rho H\right)}$$