Buckingham's π-Theorem

This method of analysis is used when number of variables are more.

Theorem:

7 ③

If there are n variables in a physical phenomenon and those n variables contain m dimensions, then variables can be arranged into (n-m) dimensionless groups called π terms.

7 - 3 = 4 $\pi_1, \pi_2, \pi_3, \pi_4,$

Explanation:

If $f(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0$ and variables can be expressed using m dimensions then $f(\pi_1, \pi_2, \pi_3, \ldots\ldots\ldots \pi_{n-m}) = 0$ where, $\pi_1, \pi_2, \pi_3, \ldots$ are dimensionless groups.

Each π term contains (m + 1) variables out of which m are of repeating type and one is of non-repeating type.

Each π term being dimensionless, the dimensional homogeneity can be used to get each π term.

π denotes a non-dimensional parameter

Buckingham's π-Theorem

$n = 7$ $\quad$ π Term $= n - m = 7 - 3 = 4$ $\quad$ m $\quad$ 1

$m = \underline{3}$

$\pi_1 \quad \underline{x_1}\ \underline{x_2}\ \underline{x_3}\ x_4$

$\pi_2 \quad x_1\ x_2\ x_3\ x_5$

$\pi_3 \quad x_1\ x_2\ x_3\ x_6$

$\pi_4 \quad x_1\ x_2\ x_3\ x_7$

$\underline{m+1}$

Selecting Repeating Variables:

1. Avoid taking the quantity required as the repeating variable.
2. Repeating variables put together should not form dimensionless group.
3. No two repeating variables should have same dimensions.
4. Repeating variables can be selected from each of the following properties.
   - Geometric property → Length, height, width, area
   - Flow property → Velocity, Acceleration, Discharge
   - Fluid property → Mass density, Viscosity, Surface tension