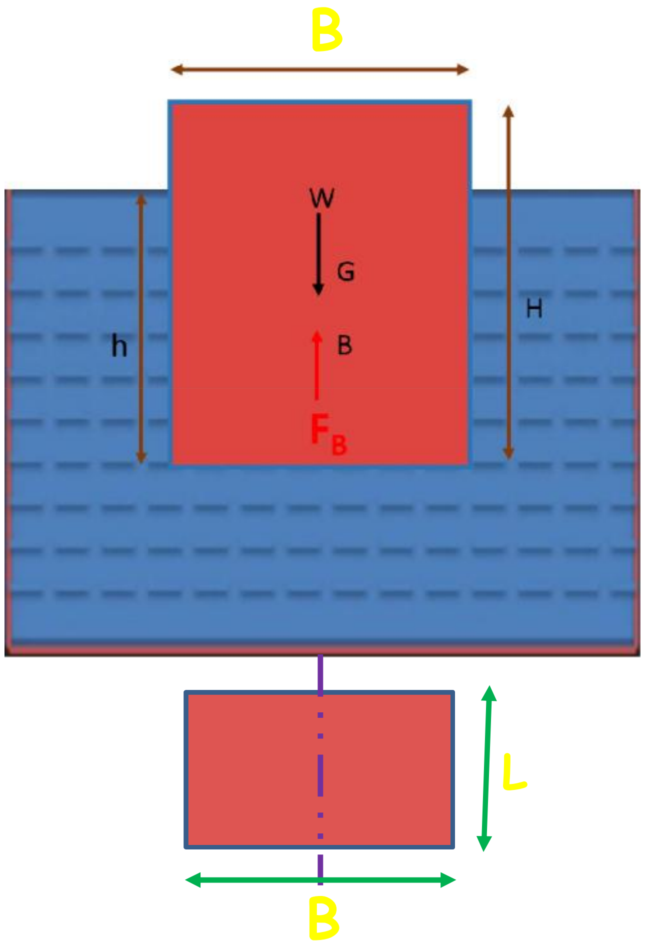
Buoyancy and Metacentric Height

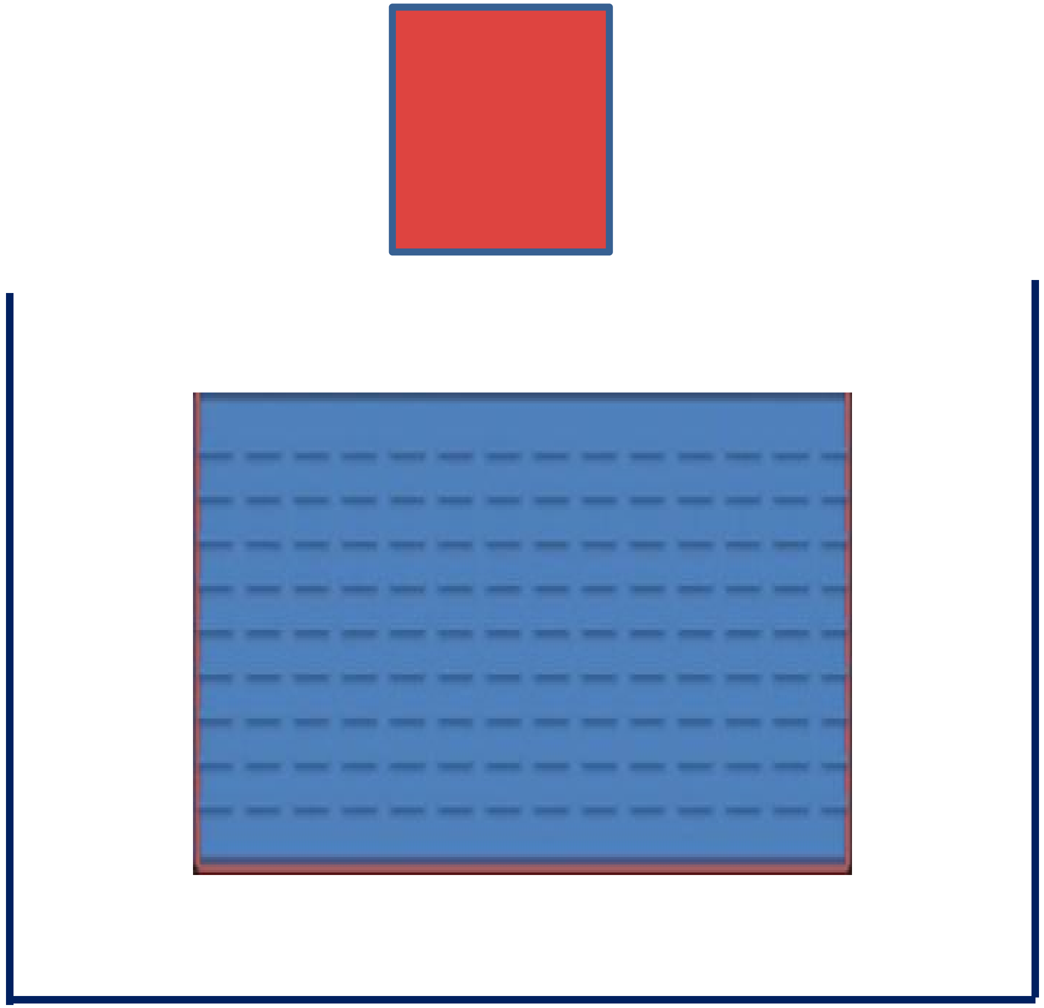
Analytical Method to calculate Metacentric Height

$$GM = BM - BG$$

$$GM = \frac{I}{V} - BG$$



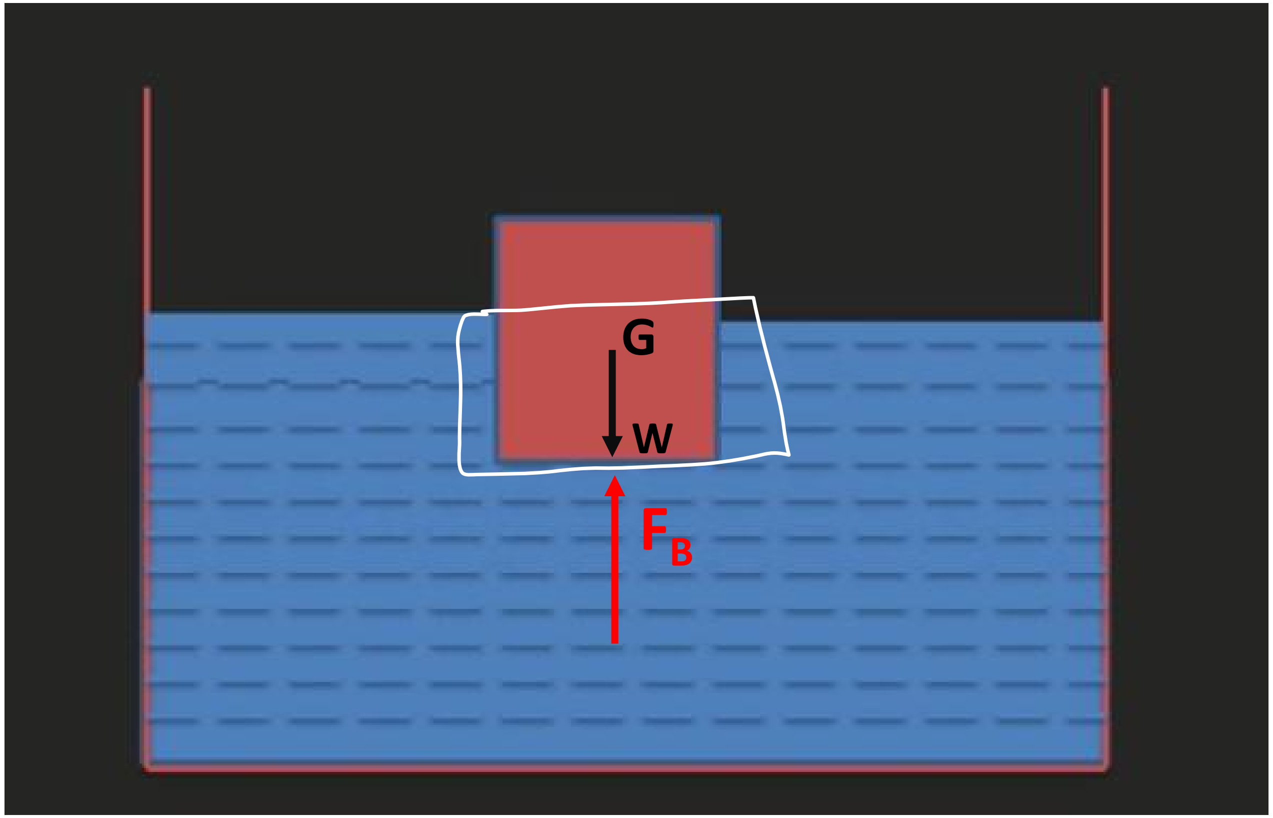
BUOYANCY



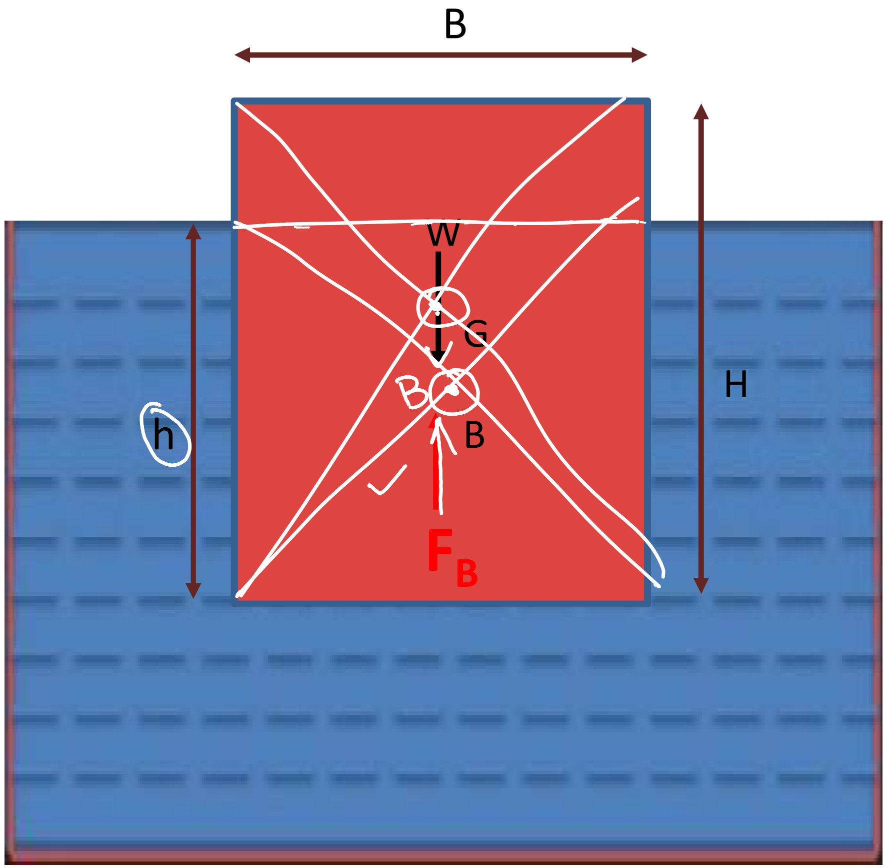
Weight of Displaced Fluid

Archimedes Principle :

Whenever a body is immersed wholly or partially in a fluid then it is lifted up by a force equal to the weight of fluid displaced by the body



Whenever a body is immersed wholly or partially in a fluid then it is lifted up by a force equal to the weight of fluid displaced by the body and this Upward force is known as Buoyancy force



F_B = Weight of water displaced by body

= m * g

= ρ * Volume of water displaced by body * g

= ρ * g * (h * B * L)

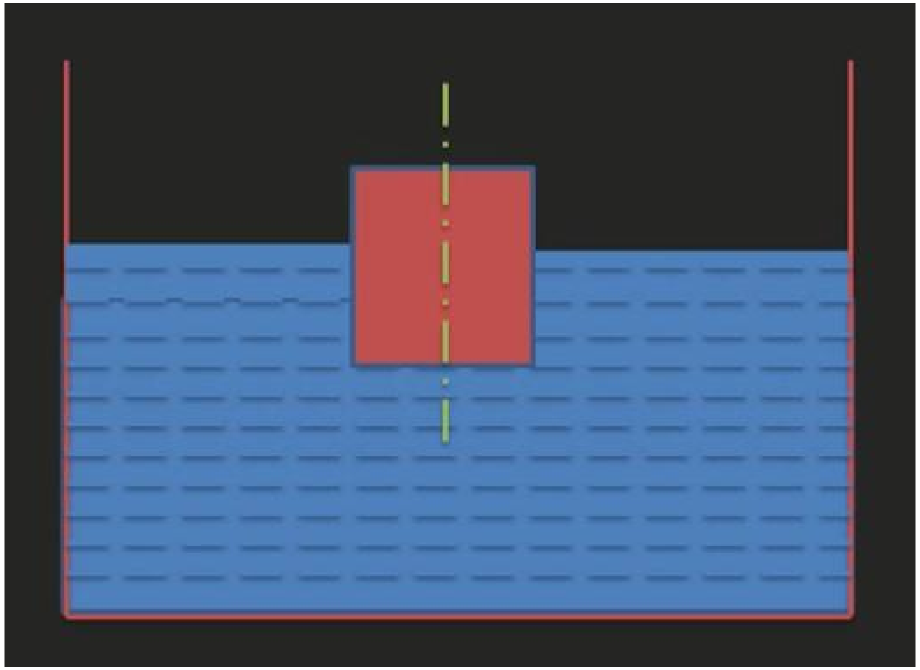
Where, h = depth of body immersed in liquid

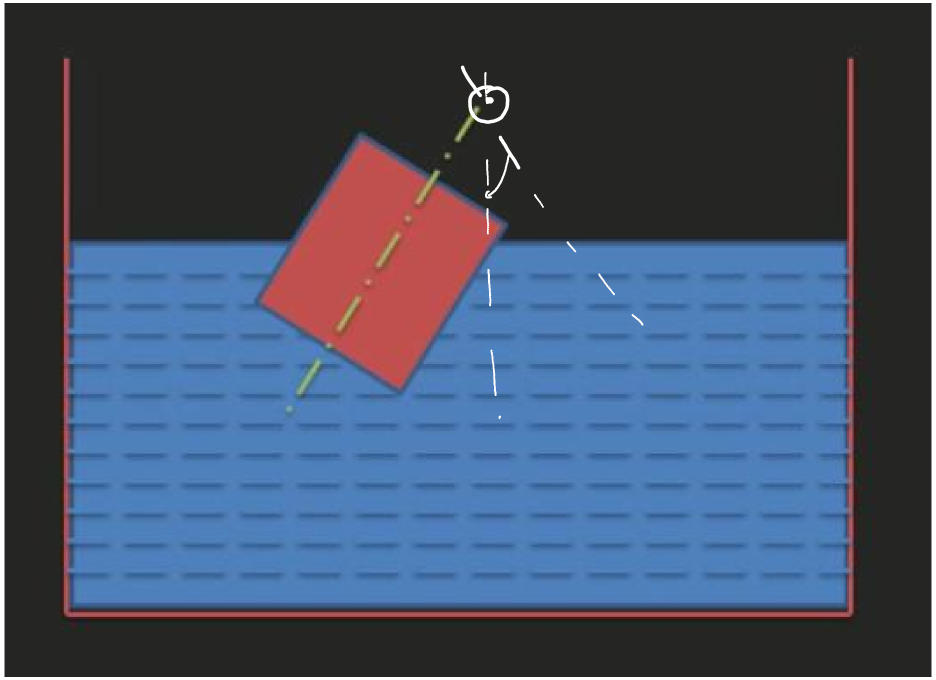
B = Width of body

L = Length of body

Meta centre

It is defined as a point with respect to which a body oscillates in a liquid, when the body is tilted through a small angle.





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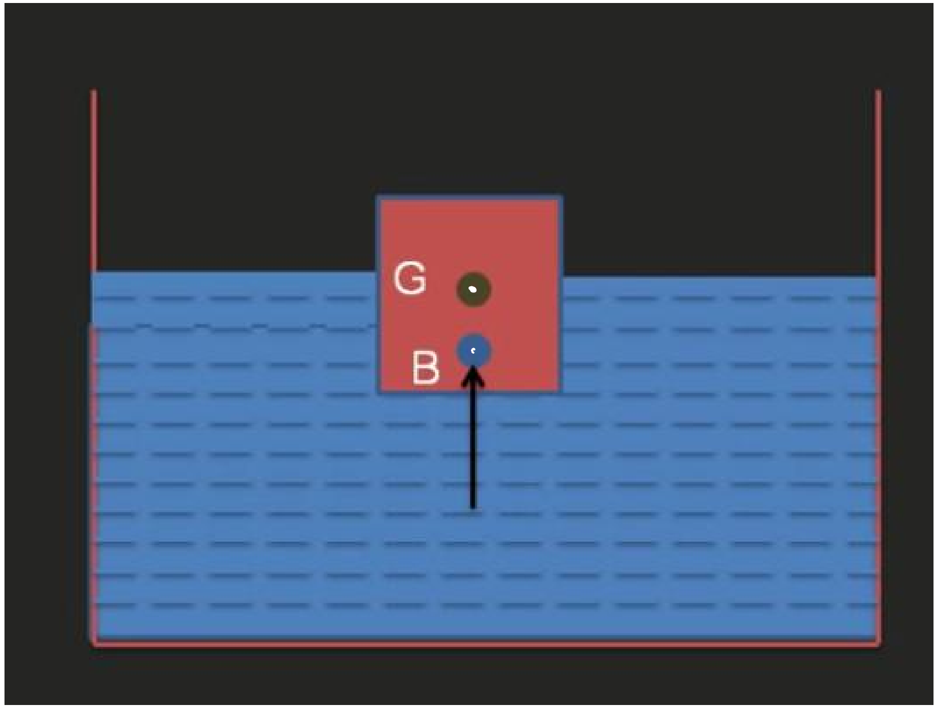
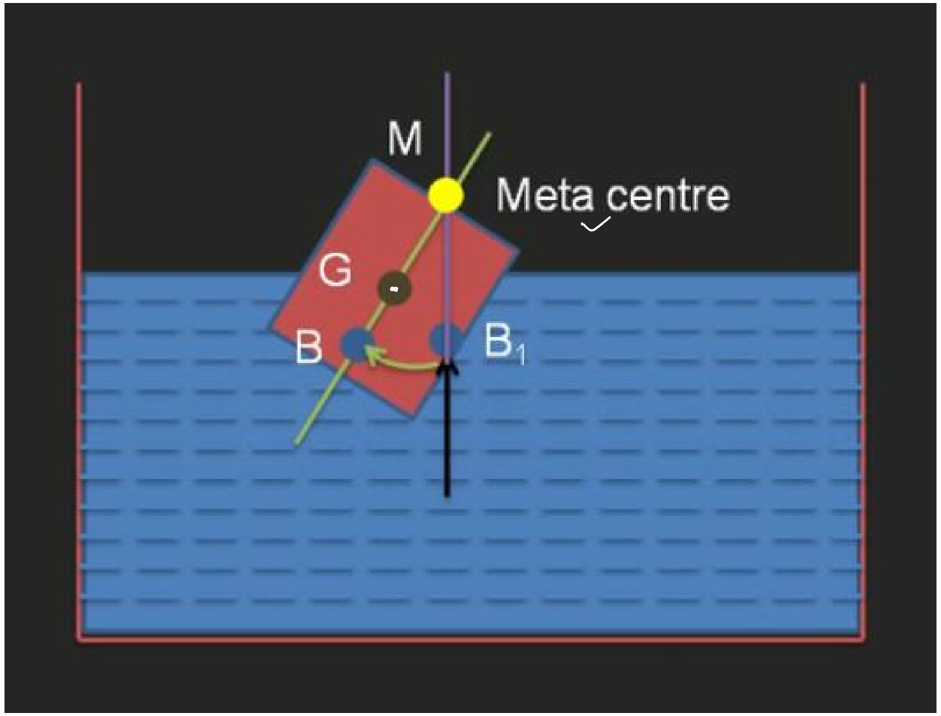
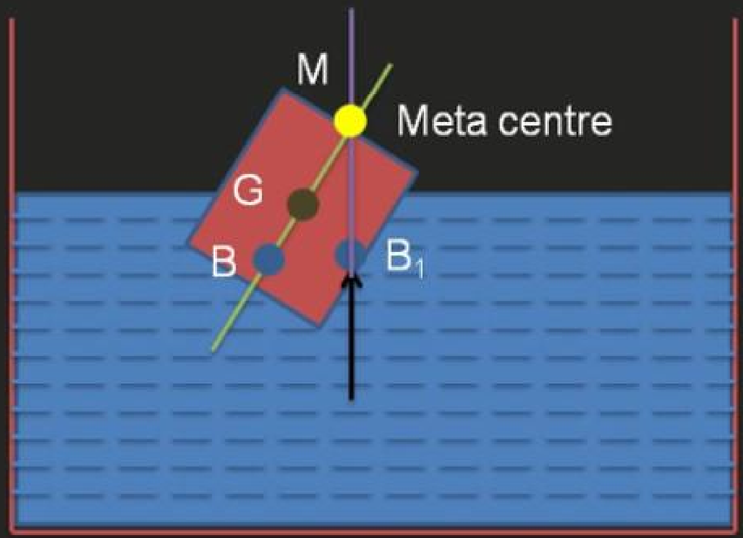




It can be also defined as an intersecting point between neutral axis line of the body and line of action of force of buoyancy.

Meta-centric height

It is the distance between meta centre and centre of gravity of the floating body.

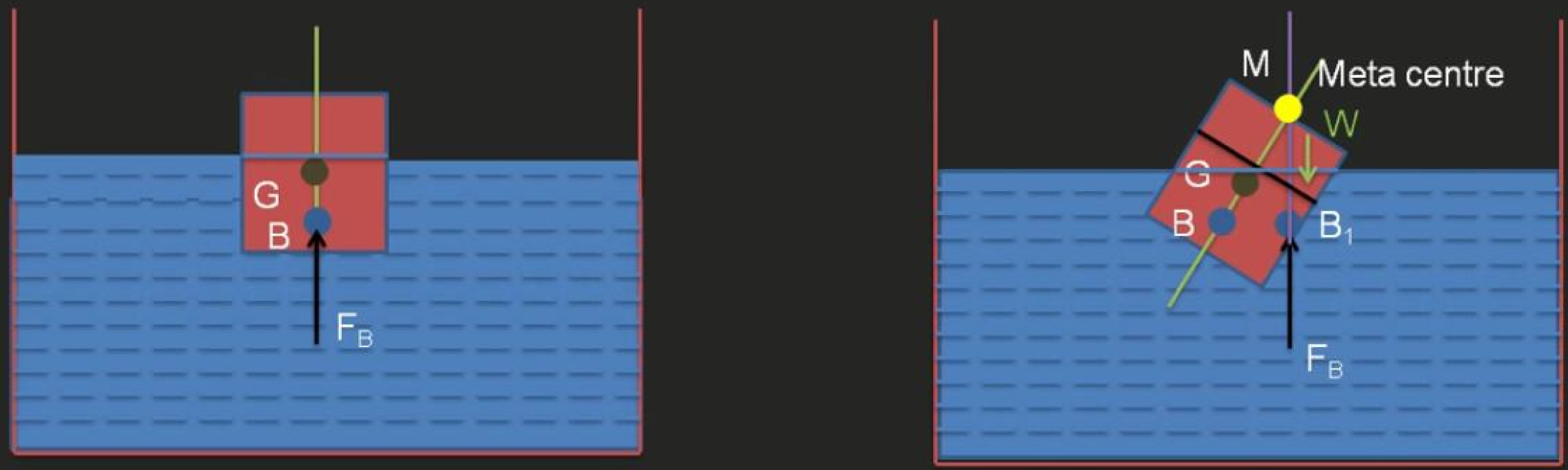


GM = Meta-centric height

$$GM = BM - BG$$

Meta-centric height

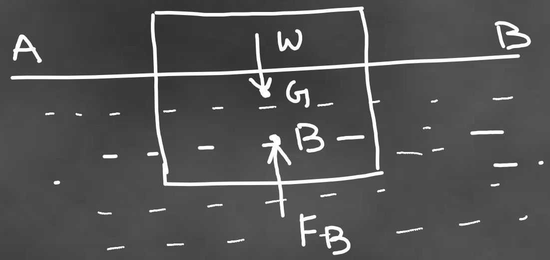
It is the distance between meta centre and centre of gravity of the floating body.

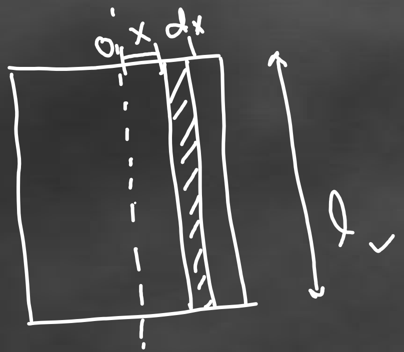


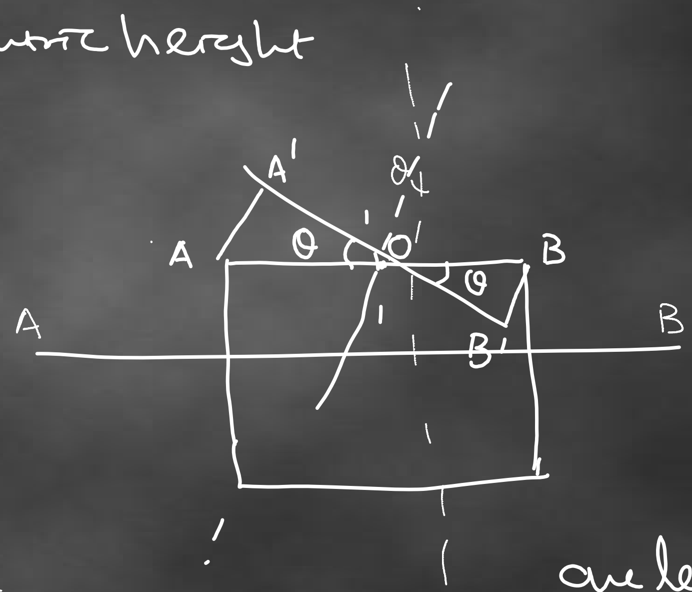
Moment due to increased weight = Moment due to buoyancy force with change of centre of buoyancy

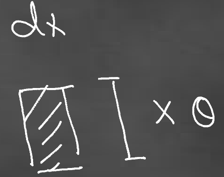
Moment = Force * Perpendicular distance

* Analytical method to find metacentric height



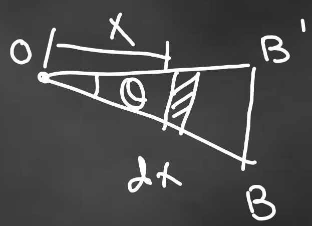


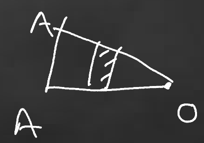




arc length $= r \times \theta$

$l = x \times \theta$





area of strip $= x\,\theta\,dx$

Volume of strip $= x\,\theta\,dx\,l$

gain of buoyancy force on right-side of strip

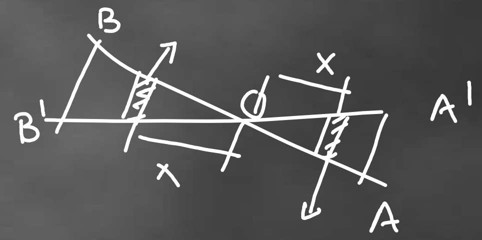
$$dF_B = \rho V g = \rho\, x\,\theta\, l\, dx\, g$$

OAA'

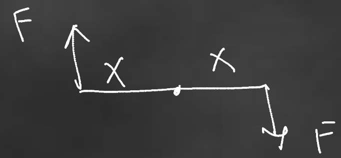
$$dF_B = \rho g x \theta l \, dx$$

moment of couple $= dF_B (x + x)$

$$= \rho g x \theta l \, dx (2x)$$



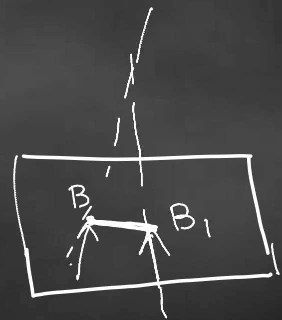
$= F \times 2x$



Moment of whole edge

$$= \int 2 \rho g \theta l x^2 \, dx$$

$F_B \times BB_1$ → cause by Displacement of Centre of buoyancy



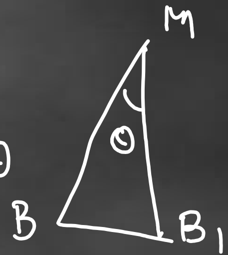
$$F_B \times BB_1 = \int 2 \rho g \theta l x^2 \, dx$$

$$\int 2\rho g \theta l x^2 dx = F_B \times BB_1$$

$$= F_B \times BM \sin\theta$$

$$\sin\theta = \frac{BB_1}{BM}$$

$$BB_1 = BM \sin\theta$$



$$(F_B = W)$$

$$W \times BM \sin\theta = \int 2\rho g \theta l x^2 dx$$

$$\sin\theta = \theta = \tan\theta$$

$$l\,dx = dA$$

$$W \times BM = \int 2\rho g\, x^2 dA$$

$$= \rho g \int 2 x^2 dA$$

$$\int 2 x^2 dA = I$$

$$W \times BM = \rho g I$$

$$W = \rho g \forall$$

$$BM = \frac{\rho g I}{\rho g \forall}$$

$$BM = \frac{I}{\forall}$$

$$BM = GM + BG$$

$$BM = \frac{I}{\forall}$$

$$BM = GM + BG$$

$$GM + BG = \frac{I}{\forall}$$

$$\boxed{GM = \frac{I}{\forall} - BG}$$

GM = Metacentric height