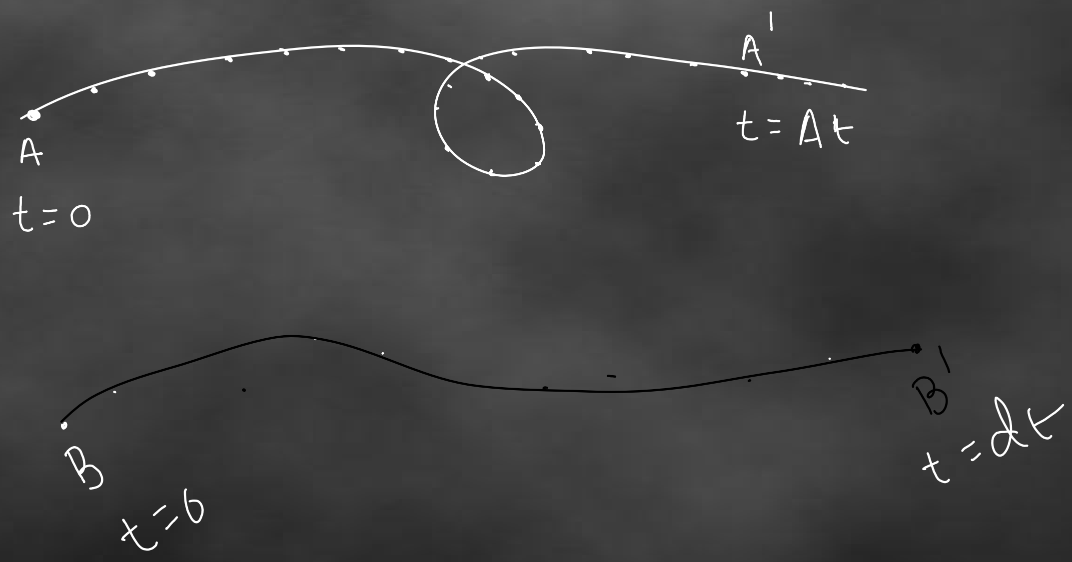
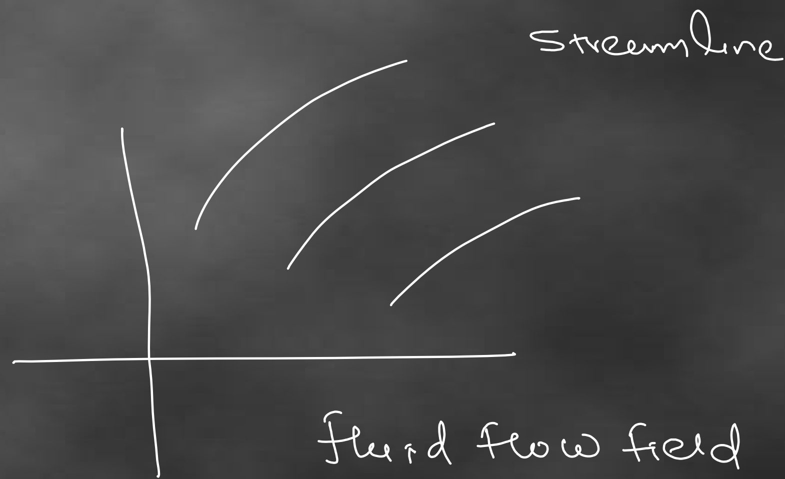
Path line

The Path Traced by a single fluid Particles in motion over a Period of time

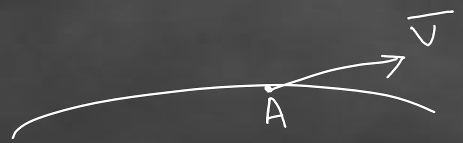


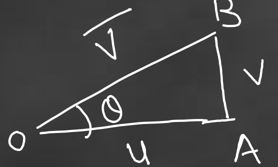
Stream Line

* An imaginary line
* at instant tangent at streamline
  local velocity vector

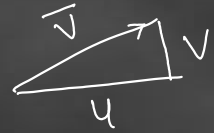


Fluid flow field





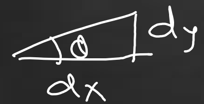
$\bar{V} \rightarrow u, v$



$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{v}{u} = \frac{dy}{dx}$$

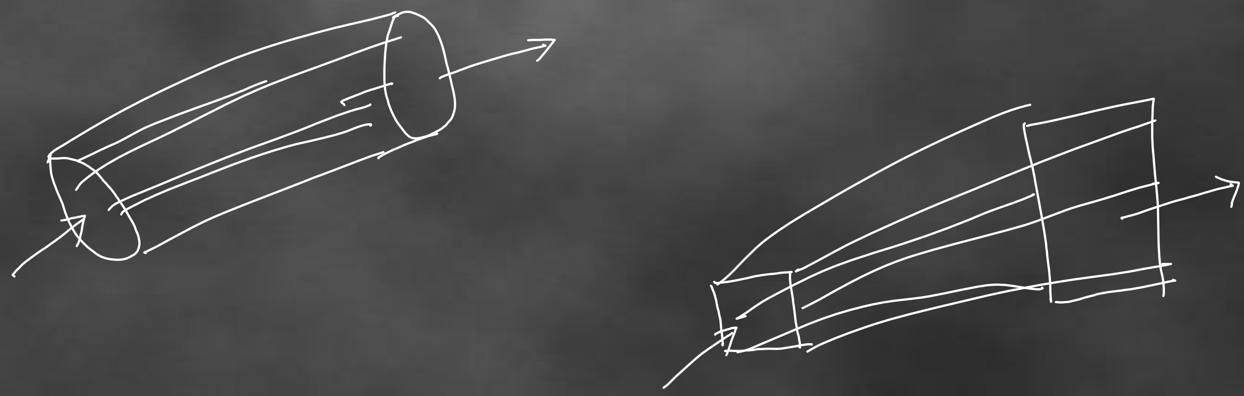
$$\tan\theta = \frac{v}{u} \Rightarrow m = \frac{dy}{dx}$$



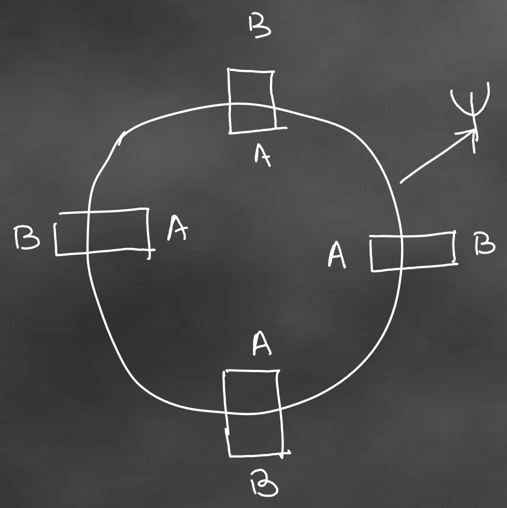
$$\frac{dx}{u} = \frac{dy}{v}$$

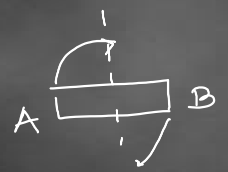
$$\boxed{v\,dx - u\,dy = 0}$$

Stream Tube



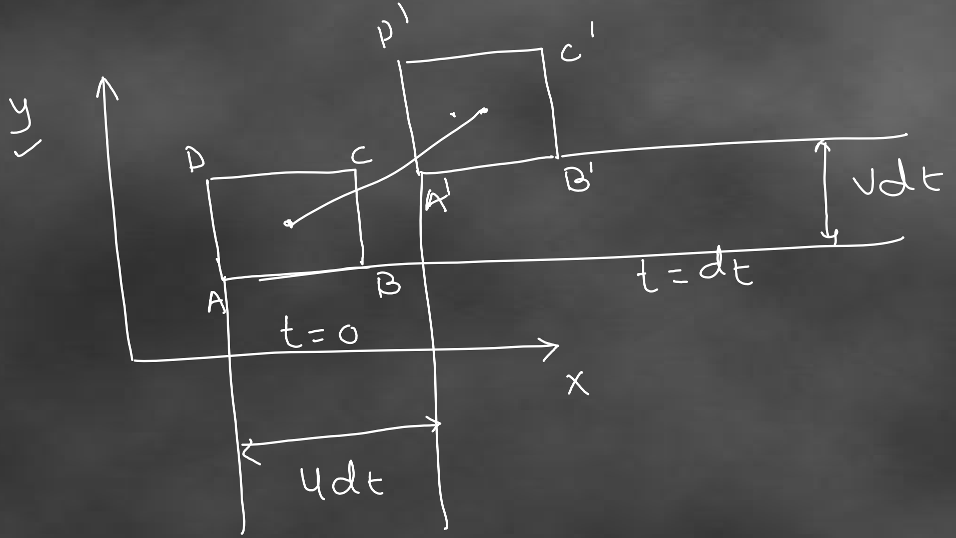
Rotational and irrotational flow





Linear Translation and Deformation

$V\,(u, v)$



$$\text{Velocity} = \frac{\text{Distance}}{\text{time}}$$

$$D = V \cdot t$$

Linear Translation

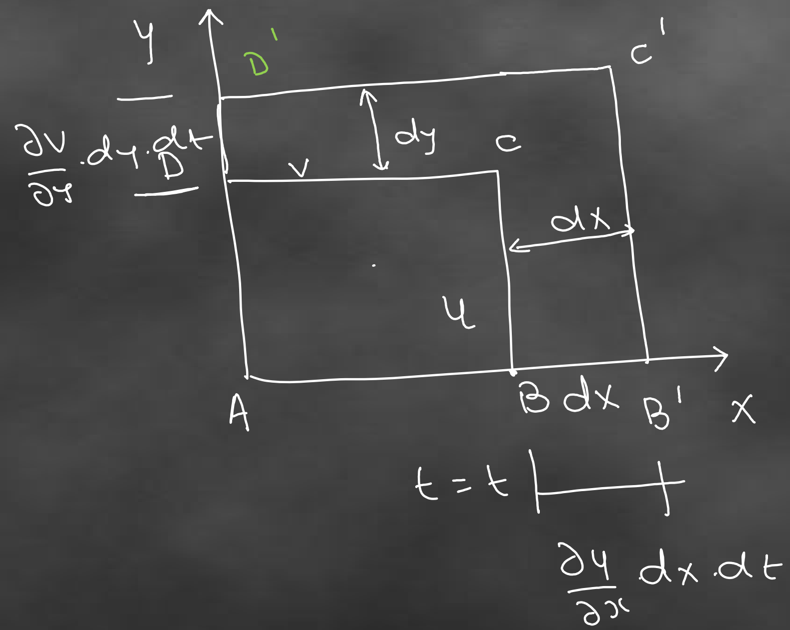
Linear Translation and Deformation

$V = \frac{d}{t}$

$d = V \cdot t$

change in velocity

$= \left(\frac{\partial v}{\partial y} \cdot dy \right)$



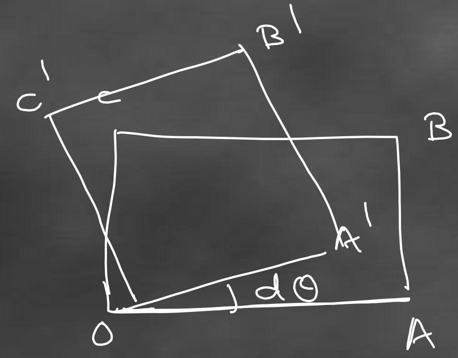
$t = t + dt$

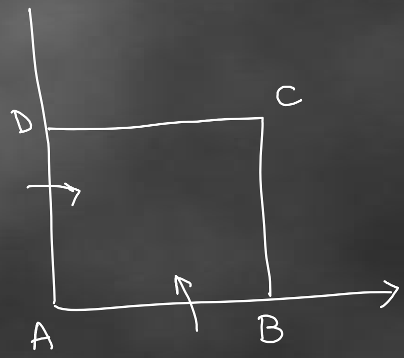
$u' = u + \frac{\partial u}{\partial x} \cdot dx$

$u' - u = \left(\frac{\partial u}{\partial x} \cdot dx \right)$

Change in velocity

Angular Translation and Deformation

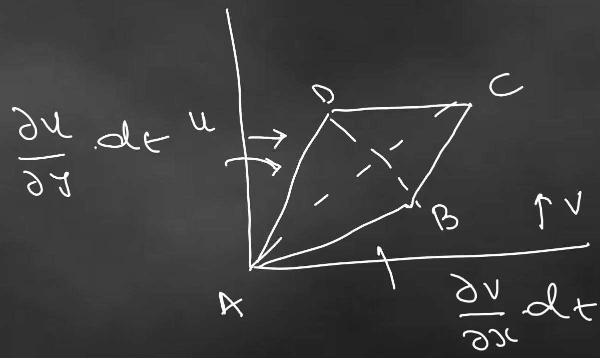




$\omega = 0$

$\omega \neq 0$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) t$$



dt

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right).$$

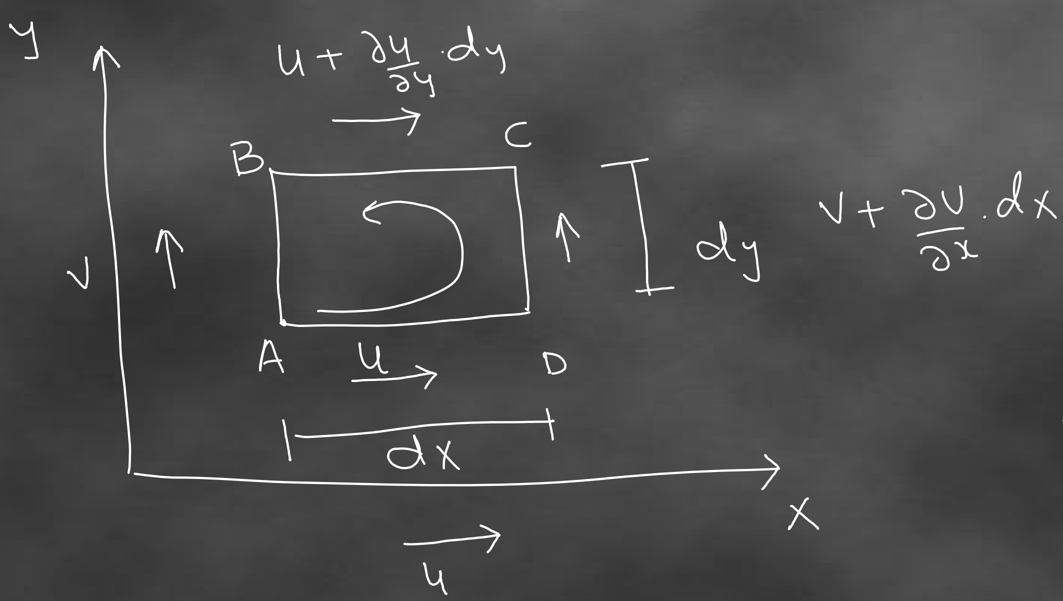
$\zeta = 2\omega$

Circulation and vorticity

$$\Gamma = \oint V \cos\alpha \, ds$$

gamma

Circulation and vorticity



$$\Gamma = \oint \underline{V \cos\alpha} \, \underline{ds}$$

$$d\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x} \cdot dx\right) dy - \left(u + \frac{\partial u}{\partial y}\, dy\right) dx - v\,dy$$

$$d\Gamma = u\,\cancel{dx} + v\,\cancel{dy} + \frac{\partial v}{\partial x} \cdot dx\,dy - \cancel{u\,dx} - \frac{\partial u}{\partial y} \cdot dx\,dy - v\,\cancel{dy}$$

$$d\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \underbrace{dx\,dy}_{A}$$

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$d\Gamma = \int \xi \, dA$$

A fluid flow is given by V=18x^3i-20x^2yj. State flow is rotational or irrotational

Given Data

$$V = 18x^3 i - 20x^2 y j$$

$$V = u i + v j$$

$$u = 18x^3 \qquad v = -20x^2 y$$

$$v = -20x^2 y \qquad \frac{\partial v}{\partial x} = -40xy$$

$$\frac{\partial u}{\partial y} = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_z = \frac{1}{2}\left(-40xy - 0\right)$$

$$\omega_z = -20xy$$

$$\omega_z \neq 0$$

Rotational ✓

Stream Function

Assumption

2-D

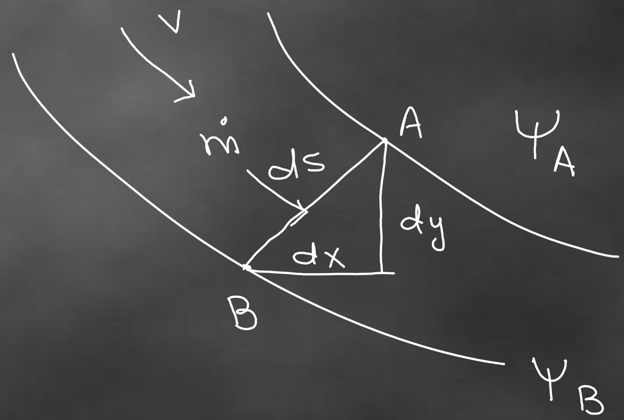
$\rho = C$

$\psi = f(x, y)$

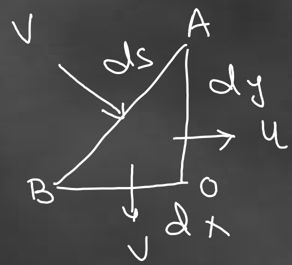
$$\frac{\partial \psi}{\partial x} = y$$

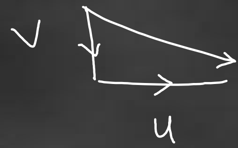
$$\frac{\partial \psi}{\partial y} = x$$

Discharge Per unit thickness of flow



flow across AB $\quad \dot{q} = A \cdot V$





$d\psi$ = flow across OA + flow across OB

$$V(ds \cdot 1) = u(dy \times 1) + -v(dx \times 1)$$

$$V ds = u\,dy - v\,dx$$

$$d\psi = u\,dy - v\,dx$$

Stream Function

$$\int d\psi = \int v\,dy - \int u\,dx \quad \text{— I}$$

$$\psi = f(x, y)$$

Total derivatives

$$d\psi = \frac{\partial \psi}{\partial x}.dx + \frac{\partial \psi}{\partial y}.dy \quad \text{— II}$$

$$v = -\frac{\partial \psi}{\partial x} \qquad u = \frac{\partial \psi}{\partial y}$$

Stream Function

1) Stream function is constant along streamline

2) Stream function for irrotational flow satisfies the laplace eqn

3) " satisfies the continuity eqn

1) Stream line

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

2-D

$$\Rightarrow \frac{dx}{u} = \frac{dy}{v}$$

$$u\,dy - v\,dx = 0$$

$$d\psi = u\,dy - v\,dx = 0$$

$$d\psi = 0 \qquad \int d\psi = 0 \Rightarrow \boxed{\psi = C}$$

Stream Function

2) $$\xi_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\psi = f(x,y)$$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

$$= \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)$$

$$= -\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

3) 2-D

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

Velocity Potential

2-D
Steady
incompressible

$\phi(x, y, t)$

$\phi(x, y)$

$\phi(x, y)$

$V = u\hat{i} + v\hat{j}$

$\frac{\partial \phi}{\partial x}$ = Velocity - x = u

$\frac{\partial \phi}{\partial y}$ = Velocity - y = v

$\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \phi}{\partial y} = v$

Characteristics Potential Function

1. For equipotential line, the Potential function is constant

Total derivatives of $\phi(x, y)$

$$\frac{\partial \phi}{\partial x} = u \quad \frac{\partial \phi}{\partial y} = v$$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$d\phi = u\,dx + v\,dy \qquad d\phi = 0$$

$$u\,dx + v\,dy = 0$$

$$\frac{dy}{dx} = -\frac{u}{v} \rightarrow \text{equipotential line}$$

Characteristics Potential Function

2. Velocity Potential satisfies the condition for irrotational flow

$$\xi_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$u = \frac{\partial \phi}{\partial x}$$

$$v = \frac{\partial \phi}{\partial y}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x} \right)$$

$$= \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x \partial y}$$

$$= 0$$

Characteristics Potential Function

2. Velocity Potential function satisfies Laplace equation

2-D Continuity eqn

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \phi}{\partial y} = v$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \rightarrow$$

Relation between stream function and Velocity Potential

$\psi = f(x, y)$

$\dfrac{\partial \psi}{\partial x} = -v \qquad v = -\dfrac{\partial \psi}{\partial x}$

$\dfrac{\partial \psi}{\partial y} = u \qquad u = \dfrac{\partial \psi}{\partial y}$

$\phi = f(x, y)$

$\dfrac{\partial \phi}{\partial x} = u \qquad u = \dfrac{\partial \phi}{\partial x}$

$\dfrac{\partial \phi}{\partial y} = v \qquad v = \dfrac{\partial \phi}{\partial y}$

$\dfrac{\partial \phi}{\partial y} = -\dfrac{\partial \psi}{\partial x}$

$\dfrac{\partial \psi}{\partial y} = \dfrac{\partial \phi}{\partial x}$ C-R

Relation between stream function and Velocity Potential

$\phi = C$ equipotential line

$\psi = C$ stream line

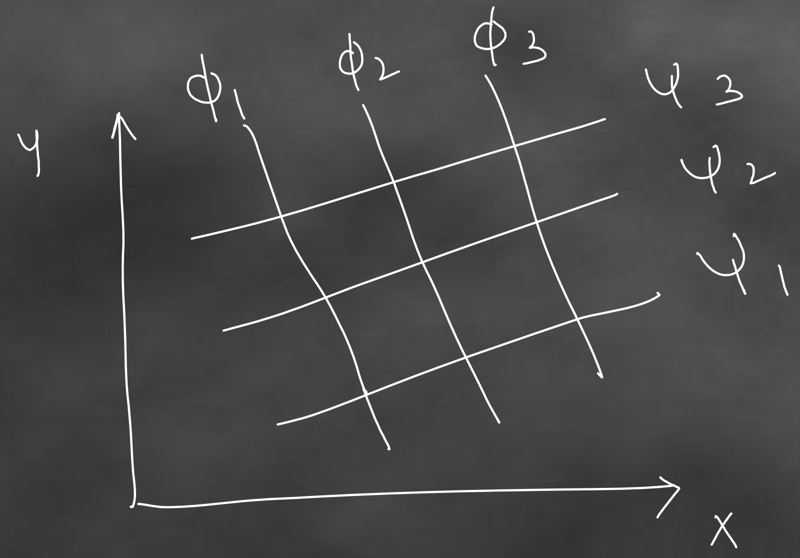
Slope $m_1 = \frac{dy}{dx} = \frac{\left(-\frac{\partial \phi}{\partial x}\right)}{\left(\frac{\partial \phi}{\partial y}\right)} = -\frac{u}{v}$

Slope $m_2 = \frac{dy}{dx} = \frac{\left(-\frac{\partial \psi}{\partial x}\right)}{\left(\frac{\partial \psi}{\partial y}\right)} = \frac{v}{u}$

$m_1 m_2 = -\frac{u}{v} \times \frac{v}{u}$

$= -1$

Relation between stream function and Velocity Potential



flow net

Q. The stream function for two dimensional flow is given by Ψ= 2xy+25, calculate the velocity at point A(2,1). Find the velocity potential function φ

given Data

$\psi = 2xy + 25$

$V = ?$ A (2,1)

$\phi = ?$

$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(2xy + 25) = 2x$

$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(2xy+25) = -2y$

$\bar{V} = ui + vj = 2xi + (-2y)j$

$\bar{V}_{(2,1)} = 4i - 2j$

$|\bar{V}| = \sqrt{4^2 + 2^2} = 4.4$

fppt.com

$$\frac{\partial \phi}{\partial x} = u = 2x \quad \text{— I}$$

$$\frac{\partial \phi}{\partial y} = v = -2y \quad \text{— II}$$

by integrating eqn I

$$\int d\phi = \int 2x\,dx$$

$$\phi = \frac{2x^2}{2} + c$$

$$\phi = x^2 + c \longrightarrow$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial c}{\partial y} = -2y$$

$$\frac{\partial c}{\partial y} = -2y \quad \text{— III}$$

$$\int dc = \int -2y\,dy = -\frac{\cancel{2}y^2}{\cancel{2}} \Rightarrow c = -y^2$$

$$\boxed{\phi = x^2 - y^2}$$

I

ψ

$\downarrow$

ϕ

II

ϕ

$\downarrow$

ψ

III

u, v

$\downarrow$

ψ

ϕ, ψ

IV

u, v

$\downarrow$

ψ

ϕ OR ψ