SUBJECT NAME : Heat Transfer

SUBJECT CODE : 3151909

Topic: Concept of Electrical analogy in Heat Transfer

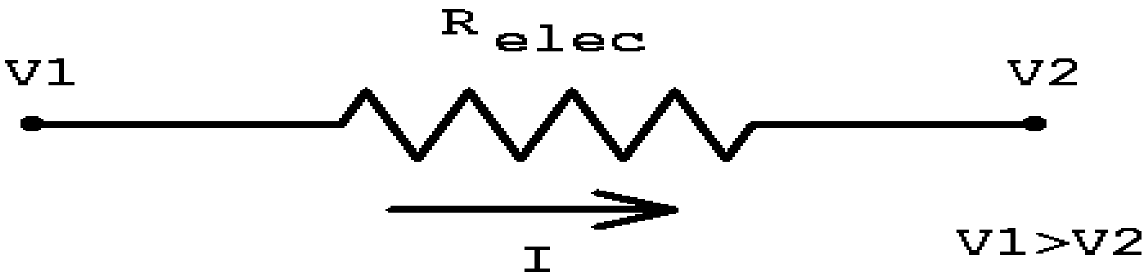
Prof. Krunal Khiraiya

Various Flows and Their Driving Forces

Flow	Driving Force
Electric Flow	Electric Potential Gradient
Fluid Flow	Pressure Gradient
Heat Flow	Temperature Gradient

Thermal resistance (Electrical Analogy)

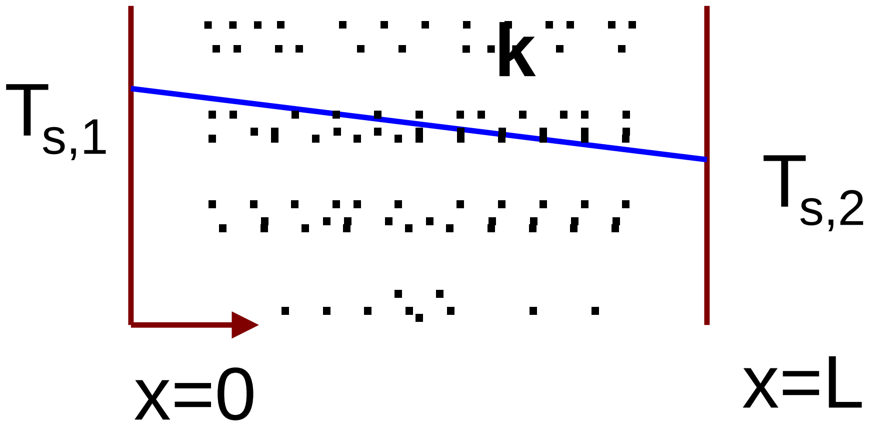
- We know that due to voltage difference, current flow through electrical circuit and the system offer some resistance to flow of current



- According to ohm's law
- **V=I X R_{elec}**
- Voltage Drop = Current flow × Resistance
- (V1-V2)/ R = I

Thermal resistance in Conduction

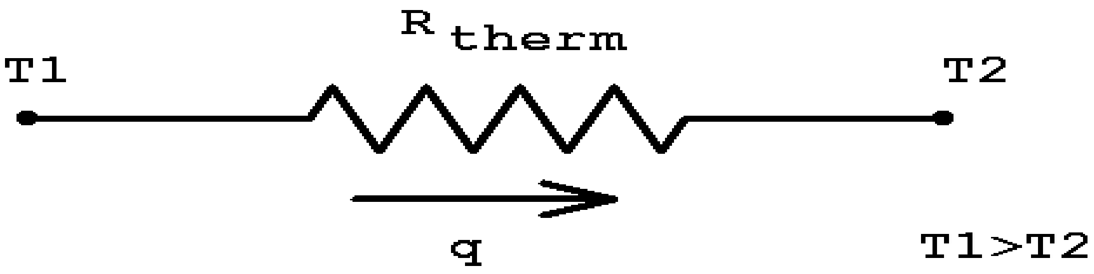
- Consider heat flow across wall /slab
- The heat flow rate across the wall is given by:



$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) = \frac{T_{s,1} - T_{s,2}}{L/kA}$$

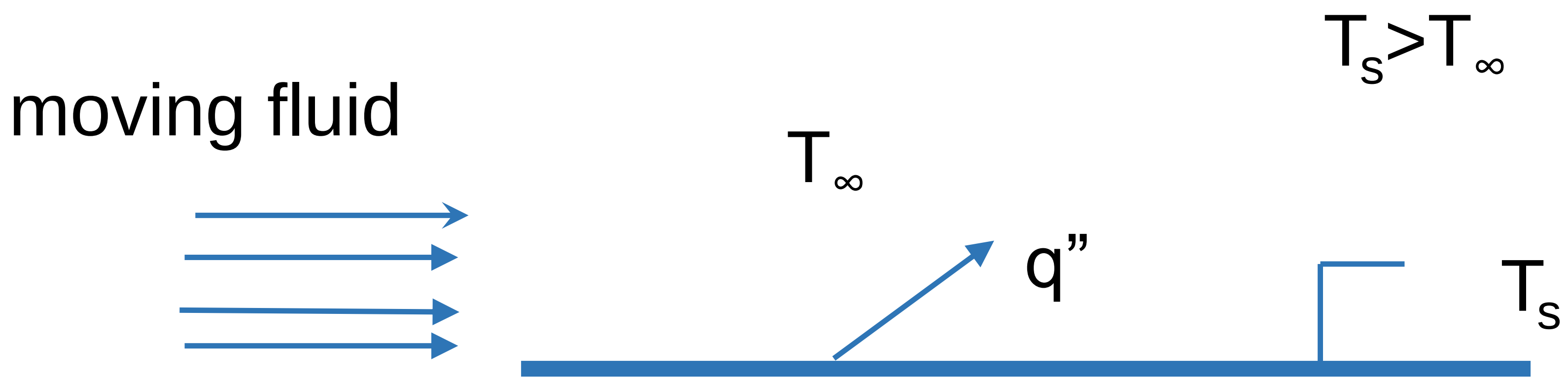
- Using electric terminology

$$\Delta T = q \times R_{therm}$$



- Temp Drop=Heat Flow × Resistance
- (T1-T2)/ Rth = Q

Thermal Resistance in Convection



- A thermal resistance may also be associated with heat transfer by convection at a surface.
- From Newton's law of cooling,

$$q = hA(T_s - T_\infty)$$

- the thermal resistance for convection is then

$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

The unit of the various parameters used

Parameter	Symbol Used	Unit
Heat Flux	q’	W/m2
Heat Flow Rate	q	W (J/s)
Thermal Conductivity	K	W/oC m
Thermal Resistance	R	oC/W

THERMAL RESISTANCES

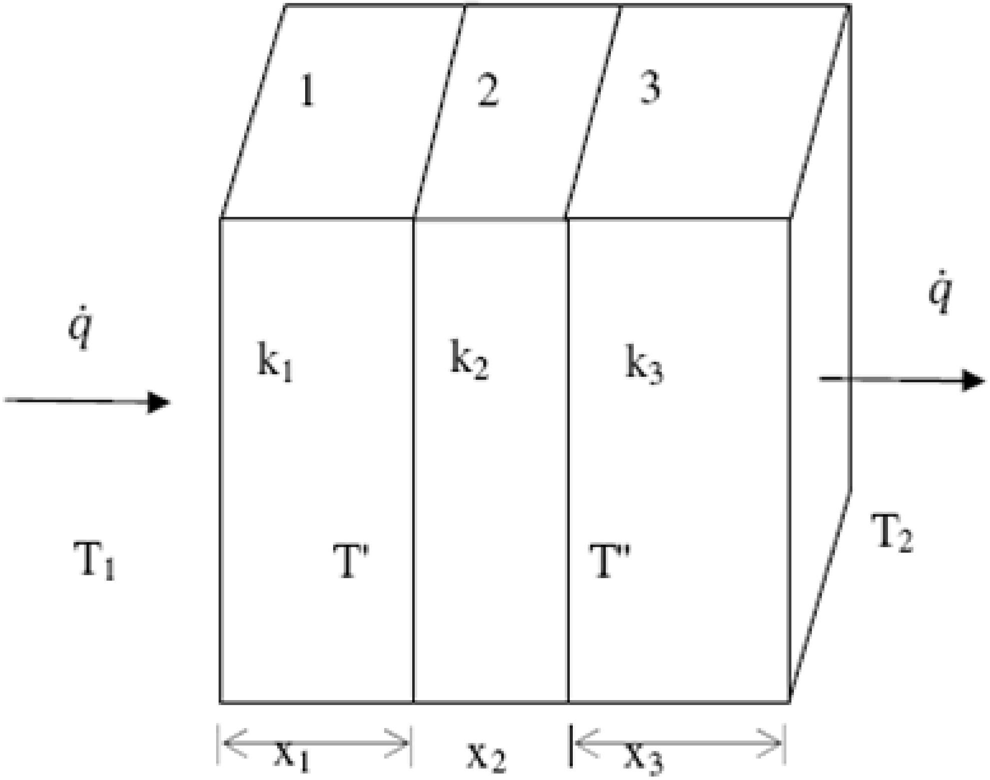
Conduction

$$R_{cond} = \Delta x/kA$$

Convection

$$R_{conv} = (hA)^{-1}$$

Heat conduction through three different layers



- Consider the area (*A*) of the heat conduction is constant and at steady state the rate of heat transfer

- from layer-1 will be equal to the rate of heat transfer from layer-2.

- Similarly, the rate of heat transfer through layer-2 will be equal to the rate of heat transfer through layer-3.

- If we know the surface temperatures of the wall are maintained at T_1 and T_2 as shown in the fig.

- the temperature of the interface of layer1 and layer 2 is assumed to be at *T'* and the interface of layer-2 and layer-3 as *T''*.

- The rate of heat transfer through layer-1 to layer-2 will be

$$\dot{q} = \frac{k_1 A\,(T_1 - T')}{x_1} \quad or \quad (T_1 - T') = \frac{\dot{q}}{1/(x_1/k_1 A)}$$

- The rate of heat transfer through layer 2 to layer 3 will be

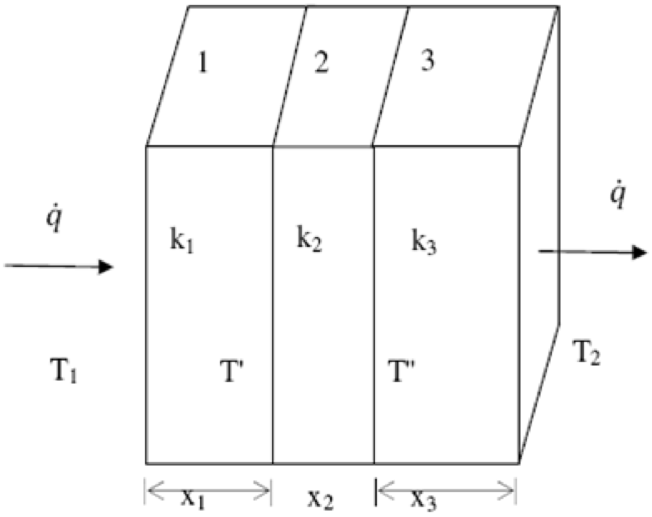
$$\dot{q} = \frac{k_2 A\,(T' - T'')}{x_2} \quad or \quad (T' - T'') = \frac{\dot{q}}{1/(x_2/k_2 A)}$$

- The rate of heat transfer through layer 3 to the other side of the wall

$$\dot{q} = \frac{k_3 A\,(T'' - T_2)}{x_3} \quad or \quad (T'' - T_2) = \frac{\dot{q}}{1/(x_3/k_3 A)}$$

- adding the above three equations

$$\dot{q} = \frac{T_1 - T_2}{\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A}} \qquad \dot{q} = \frac{T_1 - T_2}{R_1 + R_2 + R_3}$$



Equivalent electrical circuit of the three different layers

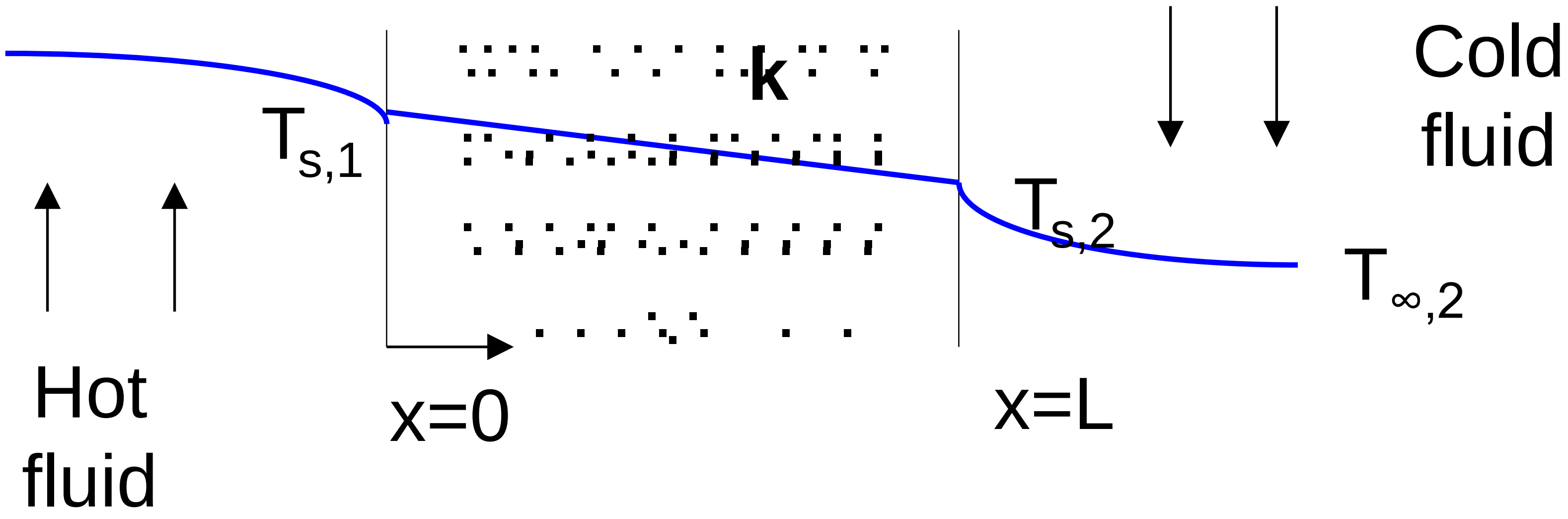
- Where, R represents the thermal resistance of the layers. The above relation can be written analogous to the electrical circuit as

$$\text{Rate of heat flow} = \frac{\text{Temperaure difference}}{\text{Resistance}}$$

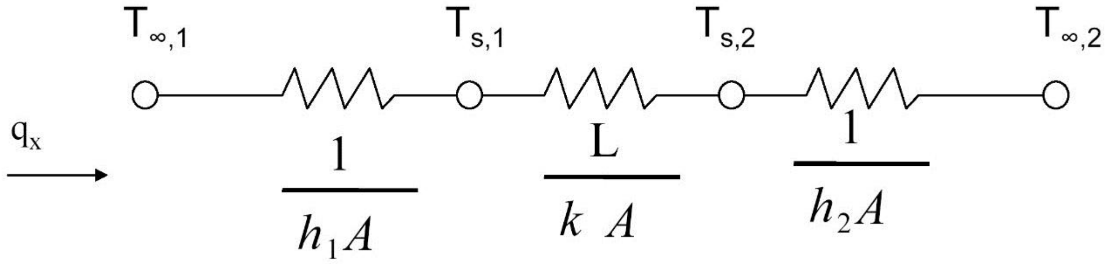
$$R_1 = \frac{x_1}{k_1 A} \qquad R_2 = \frac{x_2}{k_2 A} \qquad R_3 = \frac{x_3}{k_3 A}$$

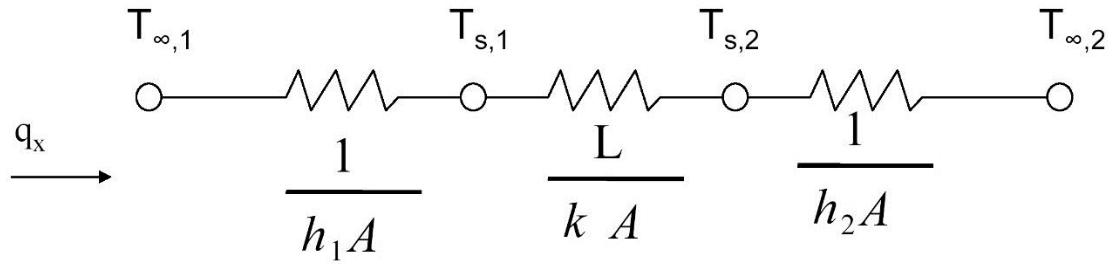
- The wall is composed of 3-different layers in series and thus the total thermal resistance was represented by R (= $R_1 + R_2 + R_3$).

Conduction through wall



- Applying thermal resistance concept to the plane wall, the equivalent thermal circuit for the plane wall with convection boundary conditions





- The heat transfer rate may be determined from separate consideration of each element in the network.
- Since q_x is constant throughout the network, it follows that

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

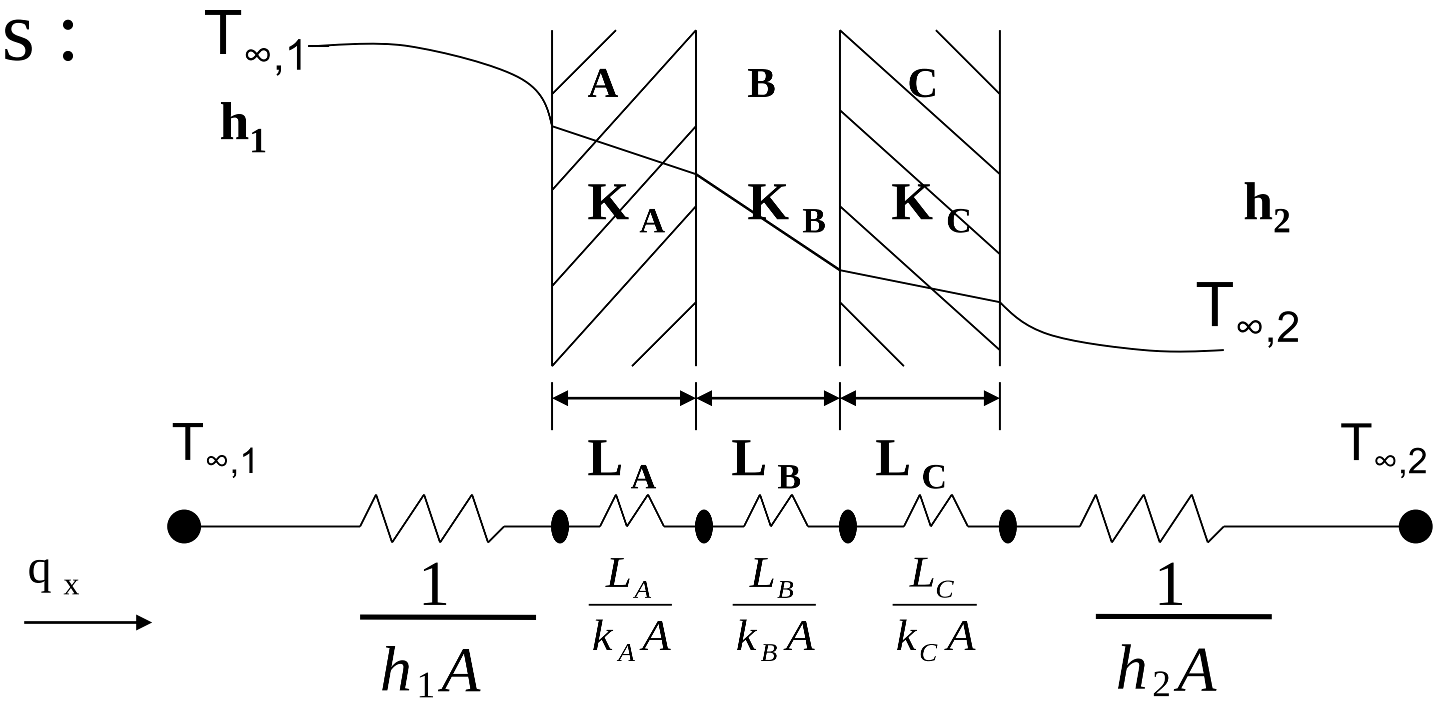
- In terms of the overall temperature difference $T_{\infty 1}$ - $T_{\infty 2}$ and
- the total thermal resistance Rtot, the heat transfer rate may also be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}$$

- Since the resistance are in series, it follows that

$$R_{tot} = \sum R_t = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

Composite Walls :

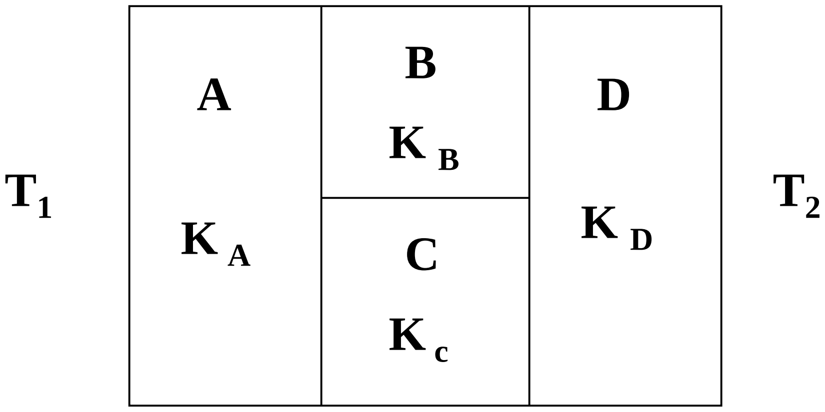


$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{\sum R_t} = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1 A} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A} + \frac{1}{h_2 A}} = UA\Delta T$$

$where, U = \frac{1}{R_{tot} A} =$ Overall heat transfer coefficient

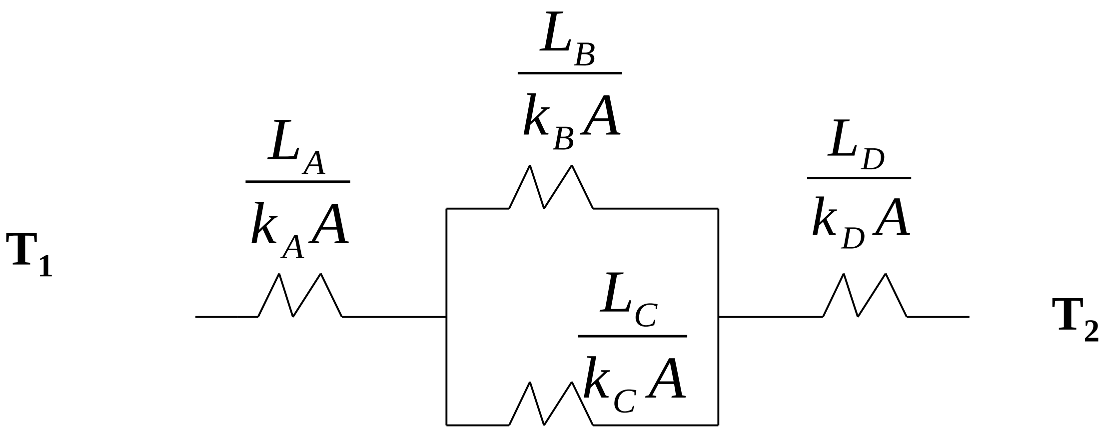
U is expressed as $$U = \frac{1}{\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}}$$

Series-Parallel :



$A_B + A_C = A_A = A_D$

$L_B = L_C$



$$q_x = \frac{T_1 - T_2}{R_{tot}}$$

$$R_{tot} = R_1 + R_{eq} + R_3$$

$$\frac{1}{R_{eq}} = \frac{1}{R_{thB}} + \frac{1}{R_{thC}}$$

$$R_{eq} = \frac{R_{thB}.R_{thC}}{R_{thB} + R_{thC}}$$