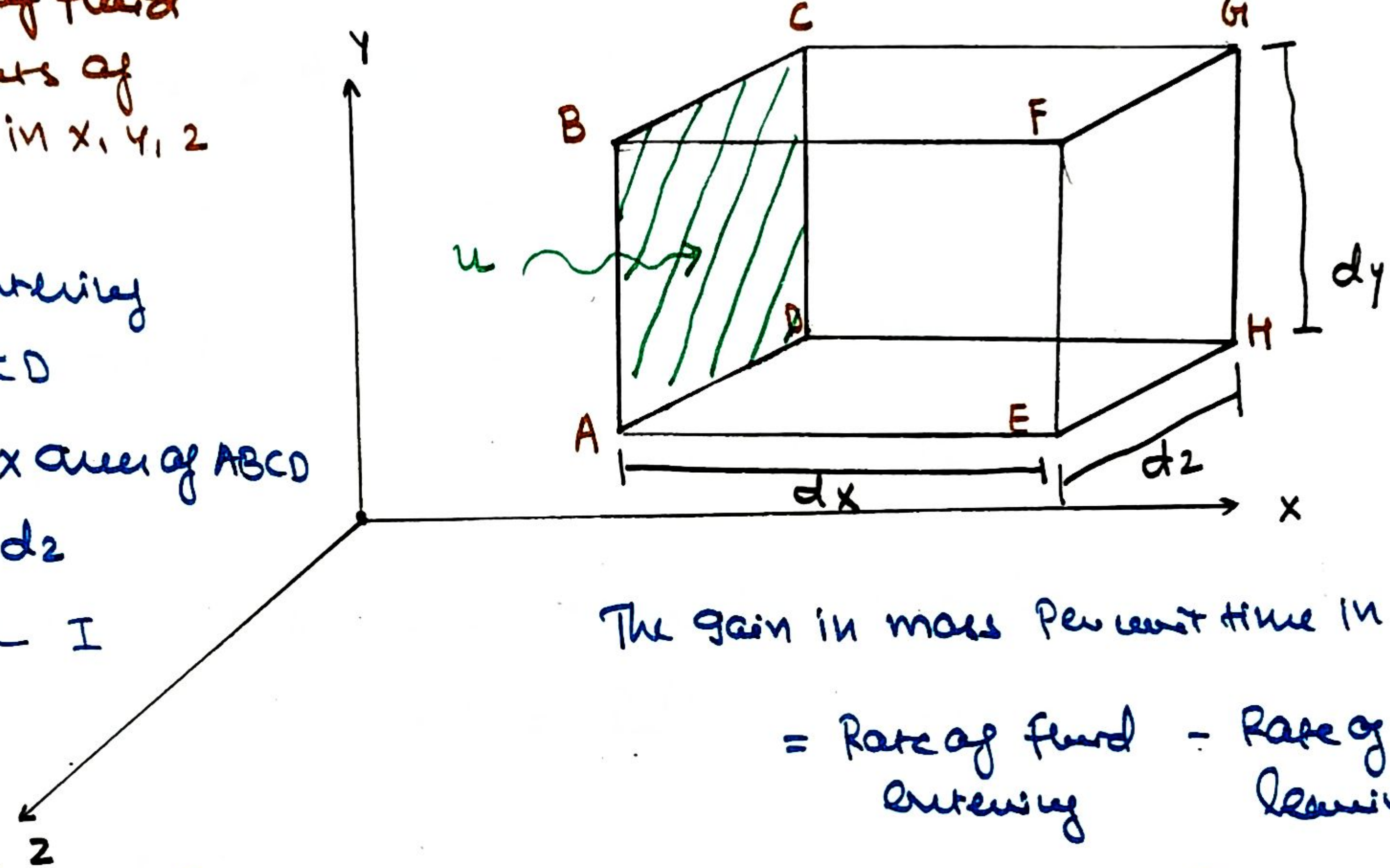
Continuity Equation

let

ρ = mass density of fluid

u, v, w = components of velocity in x, y, z



→ Rate of fluid entering from face ABCD

$= \rho \times$ velocity $\times$ area of ABCD

$= \rho \times u \times dy\,dz$

$= \rho u\,dy\,dz \quad - \mathrm{I}$

→ Rate of fluid leaving from face EFGH

$$= \rho u\,dy\,dz + \frac{\partial}{\partial x}(\rho u\,dy\,dz).dx \quad - \mathrm{II}$$

The gain in mass per unit time in x-direction

= Rate of fluid entering − Rate of fluid leaving

$$= \rho u\,dy\,dz - \left[\rho u\,dy\,dz + \frac{\partial}{\partial x}(\rho u\,dy\,dz).dx\right]$$

$$= -\frac{\partial}{\partial x}(\rho u\,dy\,dz).dx$$

$$= -\frac{\partial}{\partial x}(\rho u).dx\,dy\,dz$$

Continuity Equation

gain in mass Per unit Time in y - Direction and z - Direction

$$y\text{-Direction} = -\frac{\partial}{\partial y}(\rho v).\,dx\,dy\,dz$$

$$z\text{-Direction} = -\frac{\partial}{\partial z}(\rho w).\,dx\,dy\,dz$$

Total gain in mass Per unit time in control volume

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz \quad \text{—(III)}$$

Rate of change of mass w.r.t time in control volume

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t}(\rho V) = \frac{\partial}{\partial t}(\rho\,dx\,dy\,dz) \quad \text{—(IV)}$$

Comparing eqn III & IV

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,\cancel{dy}\,dz = \frac{\partial}{\partial t}(\rho).\,dx\,\cancel{dy}\,dz$$

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]=\frac{\partial \rho}{\partial t}$$

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}+\frac{\partial(\rho)}{\partial t}=0$$

3-D general eqn of continuity

→ Steady flow $\frac{\partial \rho}{\partial t}=0$, $\rho=c$ incompressible fluid

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

→ 2-D, steady, incompressible flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

→ 1-D steady, incompressible flow

$$\frac{\partial u}{\partial x}=0$$