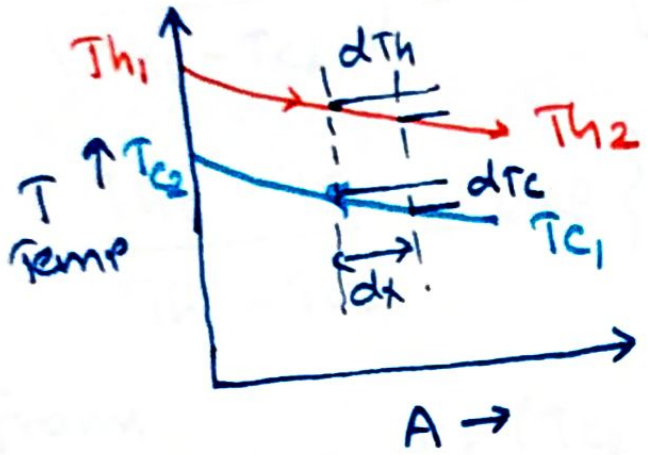
⇒ effectiveness for counter flow Heat exchanger



heat exchange $d\varphi$ under dA

$$d\varphi = U\,dA\,(T_h - T_c)$$

$$d\varphi = -m_c c_c dT_c$$

$$= -m_h c_h dT_h$$

$$dT_c = \frac{-d\varphi}{c_c}$$

$$dT_h = \frac{-d\varphi}{c_h}$$

$$d(T_h - T_c) = -\frac{d\varphi}{c_h} - \frac{d\varphi}{c_c}$$

$$= -d\varphi\left[\frac{1}{c_h} - \frac{1}{c_c}\right]$$

$$= d\varphi\left[\frac{1}{c_c} - \frac{1}{c_h}\right]$$

$$d(T_h - T_c) = U\,dA(T_h - T_c)\left[\frac{1}{c_c} - \frac{1}{c_h}\right]$$

$$\frac{d(T_h - T_c)}{T_h - T_c} = U\,dA\left[\frac{1}{c_c} - \frac{1}{c_h}\right]$$

Integrating above eqn

$$\ln(T_h - T_c) = U\,A\left[\frac{1}{c_c} - \frac{1}{c_h}\right]$$

$$\ln\left(\frac{T_{h_2} - T_{c_1}}{T_{h_1} - T_{c_2}}\right) = U\,A\left[\frac{1}{c_c} - \frac{1}{c_h}\right]$$

$$\ln\left(\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}\right) = UA\left[\frac{1}{C_c}-\frac{1}{C_h}\right]$$

$$\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}} = \exp\left\{\frac{UA}{C_c}\left[1-\frac{C_c}{C_h}\right]\right\}$$

from $$\varepsilon = \frac{C_c(T_{c2}-T_{c1})}{C_{min}(T_{h1}-T_{c1})}$$

$$\varepsilon = \frac{C_h(T_{h1}-T_{h2})}{C_{min}(T_{h1}-T_{c1})}$$

$$T_{h2} = -\frac{\varepsilon C_{min}(T_{h1}-T_{c1})}{C_h} + T_{h1}$$

$$T_{c2} = \frac{\varepsilon C_{min}(T_{h1}-T_{c1})}{C_c} + T_{c1}$$

$$\frac{-\frac{\varepsilon C_{min}(T_{h1}-T_{c1})}{C_h} + T_{h1} - T_{c1}}{T_{h1} - \left\{\frac{\varepsilon C_{min}(T_{h1}-T_{c1})}{C_c} + T_{c1}\right\}} = \exp\left\{\frac{UA}{C_c}\left(1-\frac{C_c}{C_h}\right)\right\}$$

$$\frac{(T_{h1} - T_{c1})\left[1 - \frac{\varepsilon C_{min}}{C_h}\right]}{(T_{h1} - T_{c1})\left[1 - \frac{\varepsilon C_{min}}{C_c}\right]} = \exp\left\{\frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)\right\}$$

$$\frac{1 - \varepsilon \frac{C_{min}}{C_h}}{1 - \frac{\varepsilon C_{min}}{C_c}} = \exp\left[\frac{UA}{C_c}\left(1 - C_c/C_h\right)\right]$$

let we consider $C_c < C_h$

$C_c = C_{min}$ $C_h = C_{max}$

$$\frac{1 - \frac{\varepsilon C_{min}}{C_{max}}}{1 - \frac{\varepsilon C_{min}}{C_{min}}} = \exp\left[\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right]$$

$$\frac{1 - \varepsilon \frac{C_{min}}{C_{max}}}{1 - \varepsilon} = \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}$$

$$1 - \varepsilon \frac{C_{min}}{C_{max}} = (1 - \varepsilon) \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}$$

$$1 - \varepsilon \frac{C_{min}}{C_{max}} = \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\} - \varepsilon \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}$$

$$1 - \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\} = \varepsilon\left[\frac{C_{min}}{C_{max}} - \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}\right]$$

$$\varepsilon = \frac{1 - \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}}{\frac{C_{min}}{C_{max}} - \exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\}}$$

$$\varepsilon = \frac{\exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\} - 1}{\exp\left\{\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right\} - \frac{C_{min}}{C_{max}}}$$

$$\varepsilon = \frac{\exp\left\{\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right\} - 1}{\exp\left\{\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right\} - \frac{C_{min}}{C_{max}}}$$

$$\frac{\exp\left\{\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right\} - 1}{\exp\left\{\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right\} - \frac{C_{min}}{C_{max}}}$$

$$\frac{1}{e^x} = e^{-x}$$

$$= \frac{1 - \exp\left[-\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]}{1 - \frac{C_{min}}{C_{max}}\exp\left[-\frac{UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]}$$

$$\frac{C_{min}}{C_{max}} = R \qquad \frac{UA}{C_{min}} = NTU$$

$$\varepsilon_{\text{Counter flow}} = \frac{1 - \exp\left[-NTU(1-R)\right]}{1 - R\exp\left[-NTU(1-R)\right]}$$

$$\varepsilon_{\text{Parallel}} = \frac{1 - \exp\left[-NTU(1+R)\right]}{1+R}$$

⇒ Special case

1) Condenser evaporator

$C_{max} = \infty$

$R = 0$

$$\varepsilon = 1 - e^{-NTU}$$

2) Typical Regenerator

$C_{min} = C_{max}$

$R = 1$

$$\varepsilon_{\text{Parallel}} = \frac{1 - e^{-2NTU}}{2} \qquad \varepsilon_{\text{Counter}} = \frac{NTU}{1+NTU}$$