⇒ effectiveness and Number of Transfer Unit (ε-NTU)

LMTD → Analysis of Different Type of HX
Simple equation.
Design of heat Exchanger
(Terminal Temp)

→ Temp. of fluid leaving heat exchanger unknown.
→ heat exchanger run at off Design condition.

ε - NTU → compare various Types of HX
(Selection objective)

⇒ effectiveness

$$\varepsilon = \frac{\text{actual heat Transfer}}{\text{maximum Possible heat Transfer}}$$

$$\varepsilon = \frac{q_{actual}}{q_{max}} =$$

$q_{max} = C_{min} (T_{h_1} - T_{c_1})$

max. Temp
$(T_{h_1} - T_{c_1})$

$q_{actual} = m_c c_c (T_{c_2} - T_{c_1})$
$= m_h c_h (T_{h_1} - T_{h_2})$

R = capacity Ratio

$$q_{max} = C_{min}(T_{h1} - T_{c1})$$

$$q_{actual} = C_c(T_{c2} - T_{c1})$$

$$= C_h(T_{h1} - T_{h2})$$

hot fluid

$$\varepsilon = \frac{C_h(T_{h1} - T_{h2})}{C_{min}(T_{h1} - T_{c1})}$$

cold fluid

$$\varepsilon = \frac{C_c(T_{c2} - T_{c1})}{C_{min}(T_{h1} - T_{c1})}$$

known value of

ε, T_{h1}, T_{c1}, C_{min}

$$\boxed{q = \varepsilon \, C_{min}(T_{h1} - T_{c1})}$$

Value of ε lies in the Range of 0 to 1

→ NTU

$$NTU = \frac{\text{heat capacity Rate of HX}}{\text{heat capacity rate of fluid}}$$

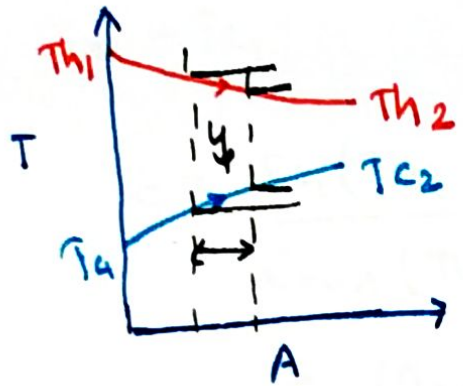
$$NTU = \frac{UA}{C_{min}}$$

U, C_{min} given value

$NTU \propto A$

NTU ↑ larger Physical size ↑

⇒ effectiveness for Parallel-flow Heat exchanger



$$dQ = U\,dA\,(T_h - T_c)$$

$$dQ = -m_h c_h \,dT_h = C_h\,dT_h$$

$$= m_c c_c \,dT_c = C_c\,dT_c$$

$$dT_h = \frac{-dQ}{C_h}$$

$$dT_c = \frac{dQ}{C_c}$$

$$d(T_h - T_c) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d(T_h - T_c) = -U\,dA\,(T_h - T_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

By integrating above eqn

$$\ln(T_h - T_c) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln(T_{h2} - T_{c2}) - \ln(T_{h1} - T_{c1}) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\left(\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = \exp\left[\frac{-UA}{C_h}\left(1 + C_h/C_c\right)\right]$$

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = \exp\left[-\frac{UA}{C_h} \cdot \left(1 + C_h/C_c\right)\right]$$

$$\varepsilon = \frac{C_h (T_{h1} - T_{h2})}{C_{min} (T_{h1} - T_{c1})}$$

$$\varepsilon = \frac{C_c (T_{c2} - T_{c1})}{C_{min} (T_{h1} - T_{c1})}$$

$$T_{h2} = \frac{\varepsilon C_{min} (T_{h1} - T_{c1})}{C_h} + T_{h1}$$

$$T_{c2} = \frac{\varepsilon C_{min} (T_{h1} - T_{c1})}{C_c} + T_{c1}$$

$$\frac{1}{T_{h1} - T_{c1}} \left[\frac{\varepsilon C_{min} (T_{h1} - T_{c1})}{C_h} + T_{h1} - \frac{\varepsilon C_{min} (T_{h1} - T_{c1})}{C_c} + T_{c1} \right] = \exp\left[-\frac{UA}{C_h} \left(1 + C_h/C_c\right)\right]$$

$$\frac{1}{T_{h1} - T_{c1}} \left[(T_{h1} - T_{c1}) - \varepsilon C_{min} (T_{h1} - T_{c1}) \left[\frac{1}{C_h} + \frac{1}{C_c} \right] \right] = \exp\left[-\frac{UA}{C_h} \left(1 + C_h/C_c\right)\right]$$

$$1 - \varepsilon C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right] = \exp\left[-\frac{UA}{C_h}\left(1 + C_h/C_c\right)\right]$$

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_h}\left(1 + C_h/C_c\right)\right]}{C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right]}$$

If

$C_c < C_h$

$C_{min} = C_c \qquad C_{max} = C_h$

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_{max}}\left(1 + \frac{C_{max}}{C_{min}}\right)\right]}{C_{min}\left(\frac{1}{C_{max}} + \frac{1}{C_{min}}\right)} = \frac{1 - \exp\left[-\frac{UA}{C_{max}}\left(1 + \frac{C_{max}}{C_{min}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}$$

$C_h < C_c$

$C_{min} = C_h$ and $C_{max} = C_c$

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}$$

$$NTU = \frac{UA}{C_{min}}$$

$$R = \frac{C_{min}}{C_{max}}$$

$$\varepsilon = \frac{1 - \exp\left[-NTU\left(1 + C_{min}/C_{max}\right)\right]}{1 + C_{min}/C_{max}}$$

$$\boxed{\varepsilon = \frac{1 - \exp\left(-NTU\left(1+R\right)\right)}{1+R}}$$