Q.1 A spherical element of 40 mm diameter is initially at temperature of 27^{0}C. It is placed in boiling water for 4 minutes. After 4 minutes, at what temperature, the sphericalelement will reach? If the same spherical element is initially at 0^{0}C, find out by lump theory that how much time will be taken by the element to reach at that temperature? Take properties of the given spherical element as: k = 10 W/m^{0}C, ρ=1200kg/m^3, c = 2KJ/kg^{0}C and heat transfer coefficient h=100W/m^{20}C

given Data

$T_\infty = 100°C$

$D = 40mm$

$R = 20mm$

$= 20 \times 10^{-3} m$

$K = 10\ w/m°C$

$\rho = 1200\ kg/m^3$

$C = 2\ KJ/kg°C$

$= 2 \times 10^3\ J/kg°C$

$h = 100\ w/m^2°C$

$L_c = \frac{V}{A_s}$

$= \frac{4/3 \pi R^3}{4 \pi R^2}$

$= \frac{R}{3}$

$L_c = \frac{20 \times 10^{-3}}{3} m$

Case - I

$T_i = 27°C \quad T_\infty = 100°C$

$T_f = ?$ $\quad \tau = 4 \times 60$

$= 240 sec$

$Bi = \frac{hL_c}{K}$

$= \frac{100 \times 20 \times 10^{-3}}{10 \times 3}$

$Bi = 0.067 < 0.1$

$$\frac{\theta}{\theta_i} = \frac{T - T_{\infty}}{T_i - T_{\infty}} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{T - 100^\circ C}{27 - 100^\circ C} = e^{\left(-\frac{100 \times 3 \times 4 \times 60}{1200 \times 20 \times 10^{-3} \times 2000}\right)}$$

$$100 - T = 16.29$$

$$\boxed{T = 83.7^\circ C}$$

Case-II

$$\overbrace{T_i = 0^\circ C \quad T_f = 83.7^\circ C}^{\tau = ?}$$

$$T_{\infty} = 100^\circ C$$

$$\frac{\theta_f}{\theta_i} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{T_f - T_{\infty}}{T_i - T_{\infty}} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{83.7 - 100}{0 - 100} = e^{\left(-\frac{100 \times 3 \times \tau}{1200 \times 20 \times 10^{-3} \times 2000}\right)}$$

$$5.88 = e^{6.25 \times 10^{-3}\tau}$$

$$\tau = 283.5 \text{ sec}$$

$$\boxed{\tau = 4 \text{ min } 43.5 \text{ sec}}$$

Q.2 During Heat Treatment cylindrical pieces of 25 mm diameter, 30 mm height and 30 ^{0}C are placed in a furnace at 750^{0}C with convection coefficient 80 w/m^{20}C. Calculate the time required to heat pieces to 600 ^{0}C. what will be shortfall in temperature if the pieces are taken out from the furnace after 280 seconds? Assume following property values density 7850 kg/m^3 specific heat 480 J/kg K conductivity 40 W/m^{0}C

Given Data

$D = 25 mm$

$h = 30 mm$

$\rho = 7850 \, kg/m^3$

$C = 480 \, J/kg\,k$

$k = 40 \, w/m^\circ c$

$h = 80 \, w/m^2 {}^\circ c$

$T_\infty = 750^\circ C$

$$l_c = \frac{V}{A_s}$$

$$= \frac{\pi r^2 h}{2\pi r(r+h)}$$

$$= \frac{rh}{2(r+h)}$$

$$= \frac{12.5\times10^{-3} \times 30\times10^{-3}}{2(12.5\times10^{-3} + 30\times10^{-3})}$$

$$= 4.41\times10^{-3} \, m$$

$$Bi = \frac{hl_c}{K}$$

$$= \frac{80\times4.41\times10^{-3}}{40}$$

$$= 0.00882$$

$$Bi < 0.1$$

Case-I

$T_i = 30^\circ C \quad T_f = 600^\circ C$

$T_{\infty} = 750^\circ C$

$$\frac{\theta_f}{\theta_i} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{T_f - T_{\infty}}{T_i - T_{\infty}} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{600-750}{30-750} = e^{\left(-\frac{80 \times \tau}{7850 \times 4.41 \times 10^{-3} \times 480}\right)}$$

$$4.8 = e^{(0.004814\,\tau)}$$

$$\tau = \frac{1.5686}{0.004814}$$

$$\boxed{\tau = 326\,sec}$$

Case-II

$T_f = ? \quad T_i = 30^\circ C \quad \tau = 280\,sec$

$T_{\infty} = 750^\circ C$

$$\frac{\theta_f}{\theta_i} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{T_f - T_{\infty}}{T_i - T_{\infty}} = e^{\left(-\frac{hA_s}{\rho V c}\tau\right)}$$

$$\frac{T_f - 750}{30-750} = \exp\left(-\frac{80 \times 280}{7850 \times 4.41 \times 10^{-3} \times 480}\right)$$

$$\exp^{(-1.348)} = \frac{T_f - 750}{30-750}$$

$$T_f = 750 + \frac{30-750}{3.849}$$

$$T_f = 563^\circ C$$

Shortfall $= 600 - 563 = \boxed{37^\circ C}$