General Heat Conduction equation in Cartesian Co-ordinate

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) + q_g = \rho . c \frac{\partial T}{\partial \tau}$$

→ Non homogeneous & anisotropic material

→ self heat generation

→ unsteady heat flow

→ Three Dimensional heat flow

$$\nabla (k \nabla T) + q_g = \rho . c . \frac{\partial T}{\partial \tau}$$

∇ - vector operator.

Case-I

isotropic material

$K_x = K_y = K_z = K$

$$\frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + q_g = \rho . C \frac{\partial T}{\partial \tau}$$

$$K\left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)\right] + q_g = \rho . C . \frac{\partial T}{\partial \tau}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{\rho . C}{K} \frac{\partial T}{\partial \tau}$$

$$\alpha = \frac{K}{\rho C} = \frac{\text{Thermal conductivity}}{\text{heat Capacity}} = \frac{W/mK}{\frac{kg}{m^3} \times \frac{J}{kg K}}$$

α = Thermal Diffusivity

$$= \frac{m^2}{sec}$$

Case-II

$q_g = 0$ No internal heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho . c}{k} \frac{\partial T}{\partial \tau}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial \tau} \quad \text{(fourier equation)}$$

Case-III Steady State heat flow

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = 0$$

$$\nabla^2 t + \frac{q_g}{k} = 0 \quad \text{(Poisson's equation)}$$

Case - IV

Steady state & no internal heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\nabla^2 t = 0 \quad \text{(Laplace eq}^n\text{)}$$

Case - V

Steady state one Dimensional

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = 0$$

Case - VI

Steady state one Dimensional, no internal heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Case - VII

Steady State Two Dimensional

no heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Case - VIII

Unsteady state, one - Dimensional,

no internal heat generation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$