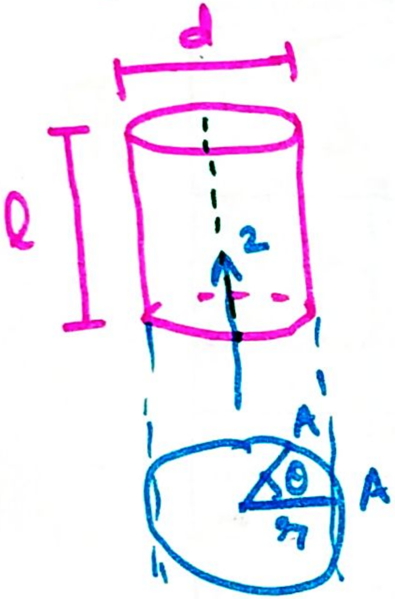
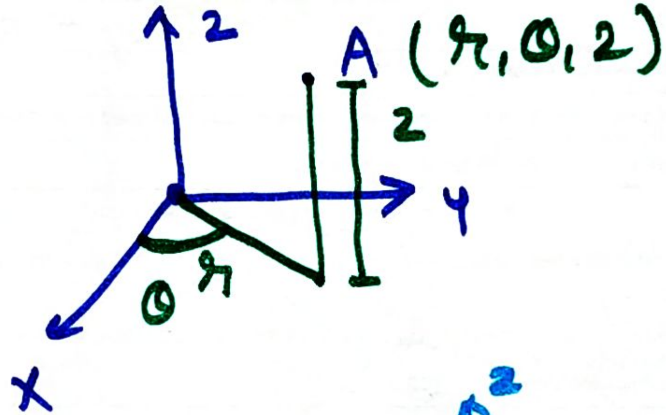
general Heat conduction eqn in cylindrical co-ordinate system.

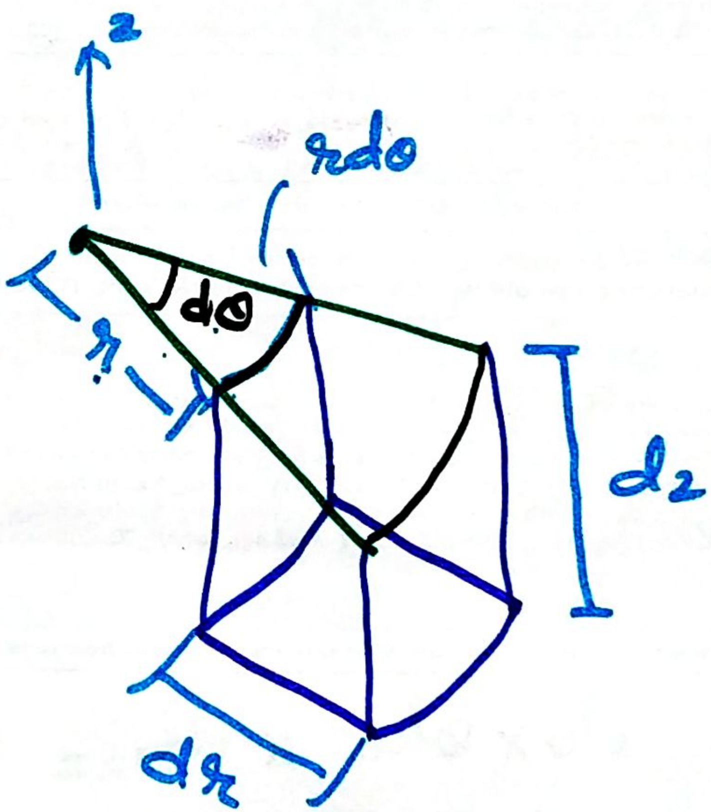
Pipe Rod cylinder geometry (r, θ, z)

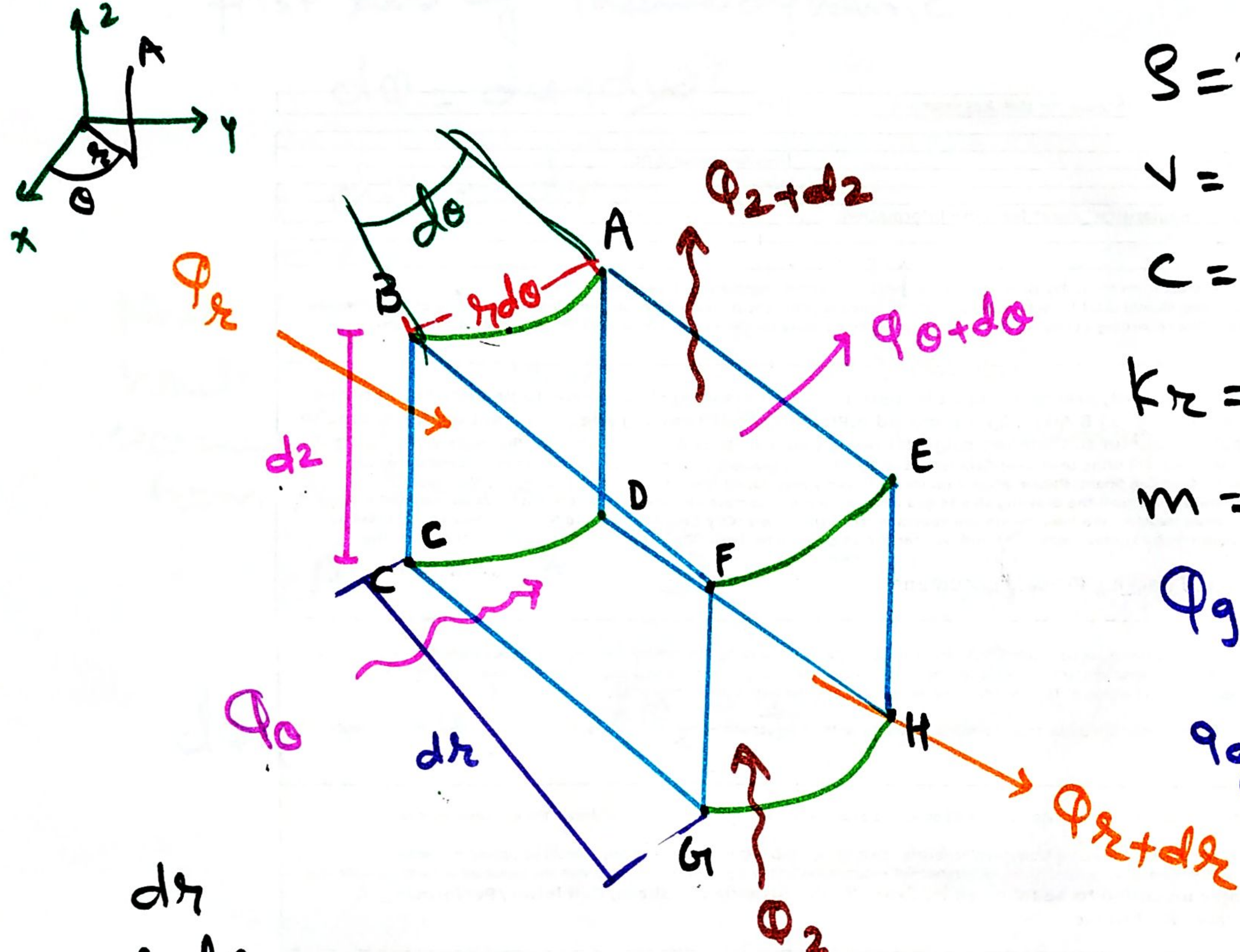




$l = r \times \theta$

$(r d\theta, dr, dz)$





ρ = Density

V = Volume

C = Specific heat

$k_r = k_\theta = k_z = k$

m = mass of element

Q_g = heat generation in unit time

q_g = heat generated / unit volume / time

dr

$r\,d\theta$

dz

Volume of element = $dr \times r\,d\theta \times dz$

first law of Thermodynamic

$$dQ = du + \cancel{dw}$$

$$dQ = du$$

Net heat accumulate from All Direction = internal heat generation = heat stored in volume

A + B = C

$$dQ'_r + dQ'_\theta + dQ'_z +$$

heat enter from plane ABCD

$$q_r = -k_r\,(r\,d\theta\,.\,dz)\,.\,\frac{\partial T}{\partial r}.$$

$$q'_r = q_r\,.\,\partial z$$

$$q'_r = -k_r\,(r\,d\theta\,.\,dz)\,\frac{dT}{\partial r}\,.\,dz$$

$$q'_{r+dr} = q'_r + \frac{\partial}{\partial r}(q'_r)\,.\,dr$$

Net heat accumulated from r-Direction

$$dq'_r = q'_r - q'_{r+dr}$$

$$= q'_r - \left[q'_r + \frac{\partial}{\partial r}(q'_r)\,.\,dr\right]$$

$$= \cancel{q'_r} - \cancel{q'_r} - \frac{\partial}{\partial r}(q'_r)\,.\,dr$$

$$dq'_r = -\frac{\partial}{\partial r}(q'_r).dr$$

$$= -\frac{\partial}{\partial r}\left(-K_r.(r d\theta.dz).\frac{\partial T}{\partial r}.\partial z\right).dr$$

$$= \frac{\partial}{\partial r}\left(K_r.r\frac{\partial T}{\partial r}\right) dr\, d\theta .dz .dz$$

$$\boxed{dq'_r = K\frac{\partial}{\partial r}\left(r.\frac{\partial T}{\partial r}\right).dr.d\theta.dz.dz}$$

Now heat accumulated in θ-Direction

$$q'_\theta = -K_\theta(dr.dz)\frac{\partial T}{r\partial\theta}.\partial z$$

$$q'_{\theta+d\theta} = q'_{\theta} + \frac{\partial}{r\partial\theta}(q'_{\theta}) \cdot r d\theta$$

Net heat accumulated

$$dq'_{\theta} = q'_{\theta} - q'_{\theta+d\theta}$$

$$= q'_{\theta} - \left[q'_{\theta} + \frac{\partial}{r\partial\theta}(q'_{\theta}) \cdot r d\theta \right]$$

$$= -\frac{\partial}{r\partial\theta}(q'_{\theta}) \cdot r d\theta$$

$$dq'_{\theta} = \frac{\partial}{r\partial\theta}\left[(K_{\theta} \cdot dr\, dz) \frac{\partial T}{r\partial\theta} \right] \partial z \; r d\theta$$

$$\boxed{dq'_{\theta} = K\,(dr\, d\theta\, dz) \frac{1}{r^2} \frac{\partial}{\partial\theta}\left(\frac{\partial T}{\partial\theta}\right) r \partial z}$$

Net heat accumulated in z - Direction.

heat influx

$$q'_z = -k_z \, (dr . r d\theta) \frac{\partial T}{\partial z} . \partial z$$

$$q'_{z+dz} = q'_z + \frac{\partial}{\partial z}(q'_z) . dz$$

$$dq'_z = q'_z - q'_{z+dz}$$

$$= q'_z - \left[q'_z + \frac{\partial}{\partial z}(q'_z) . dz \right]$$

$$= \cancel{q'_z} - \cancel{q'_z} - \frac{\partial}{\partial z}(q'_z) . dz$$

$$= - \frac{\partial}{\partial z}(q'_z) . dz$$

$$dQ'_z = -\frac{\partial}{\partial z}(Q'_z).dz$$

$$= -\frac{\partial}{\partial z}\left(-k_z\, r dr d\theta . \frac{\partial T}{\partial z}.\partial\tau\right).dz$$

$$\boxed{dQ'_z = k\; r dr d\theta dz . \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right).\partial\tau}$$

B) Heat Generated

$$Q'_g = q_g \times \text{Volume} \times \text{Time}$$

$$= q_g \times (r dr d\theta dz) \times \partial\tau$$

c) energy Stored

$$C = mc\frac{dT}{\partial\tau}\partial\tau$$

$$= \rho c(r dr d\theta dz)\frac{\partial T}{\partial\tau}.\partial\tau$$

$$A + B = C$$

$$dq'_r + dq'_\theta + dq'_z + q'_g = \text{heat stored.}$$

$$k\, dr\, d\theta\, dz\, dz \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial T}{\partial r}\right) + k\, r\, dr\, d\theta\, dz\, dz \cdot \frac{1}{r^2}$$

$$\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + k\, r\, dr\, d\theta\, dz\, dz \cdot \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)$$

$$+ q_g\, r\, dr\, d\theta\, dz\, dz = \rho c\, r\, dr\, d\theta\, dz\, \partial z \frac{\partial T}{\partial z}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \cdot \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q_g}{K} = \frac{\rho c}{K}\frac{\partial T}{\partial z}$$

$$\frac{1}{r}\left(\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{\rho c}{K}\frac{\partial T}{\partial z}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{\rho . c}{K}\frac{\partial T}{\partial z}$$