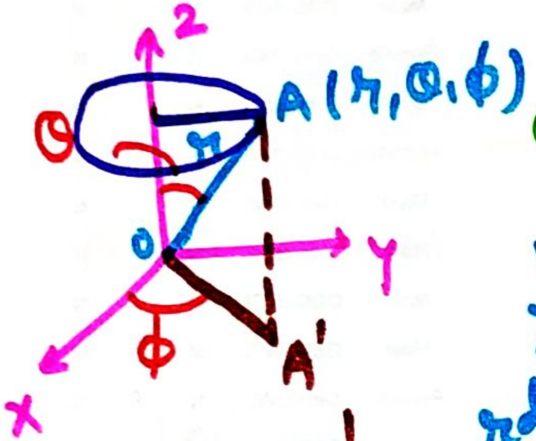
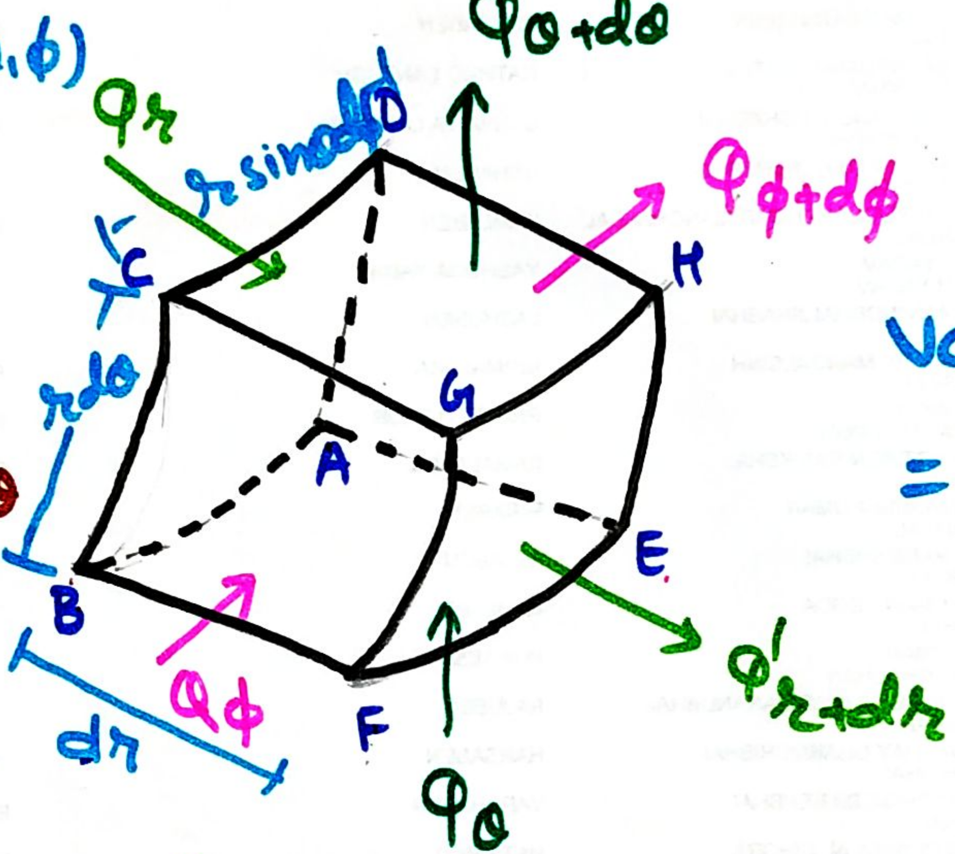
general Heat Conduction eqn in spherical co-ordinate

A (r, θ, ϕ)



OA' = $r \sin\theta$



dr

$r d\theta$

$r \sin\theta\, d\phi$

Volume of element $= dr \cdot r d\theta \cdot r \sin\theta\, d\phi$

q_g = heat gen. / unit time / unit volume

$l = r \times \theta$

let m = mass of element

ρ = Density

C = Specific heat

K = Thermal Conductivity

Now $A + B = C$

Net heat accumulated from All Direction in element + internal heat generated = heat stored in element

$$\frac{dq'_r}{}$$

dq'_θ

dq'_ϕ

Now,

$$q'_\phi = -k\,(dr.\,r\,d\theta)\,\frac{\partial T}{r\sin\theta\,d\phi}\,.\,\partial z$$

$$q'_{\phi+d\phi} = q'_\phi + \frac{\partial}{r\sin\theta\,\partial\phi}(q'_\phi).\ r\sin\theta.d\phi$$

Net heat accumulated in ϕ-Direction

$$dq'_\phi = q'_\phi - q'_{\phi+d\phi}$$

$$= -\frac{\partial}{r\sin\theta\,\partial\phi}\left(-k\ dr.\,r\,d\theta\ \frac{\partial T}{r\sin\theta\,\partial\phi}\,.\,\partial z\right).\,r\sin\theta\,d\phi$$

$$= (k\ dr\ r\,d\theta\ r\sin\theta\,d\phi)\ \frac{1}{r^2\sin^2\theta}\ \frac{\partial}{\partial\phi}\left(\frac{\partial T}{\partial\phi}\right)\partial z$$

$$= k\,dr.\ r\,d\theta\ r\sin\theta\,d\phi\ \frac{1}{r^2\sin^2\theta}\ \frac{\partial^2 T}{\partial\phi^2}\,.\,\partial z$$

Heat flow θ - Direction

$$q'_\theta = -k\,(dr \cdot r\sin\theta\, d\phi)\,\frac{\partial T}{r\,\partial\theta} \cdot \partial z$$

$$q'_{\theta+d\theta} = q'_\theta + \frac{\partial (q'_\theta)}{r\,\partial\theta} \cdot r\,d\theta$$

$$dq'_\theta = q'_\theta - q'_{\theta+d\theta}$$

$$= -\frac{\partial}{r\,\partial\theta}(q'_\theta) \cdot r\,d\theta$$

$$= k\,\frac{\partial}{r\,\partial\theta}\left(dr \cdot r\sin\theta\, d\phi \cdot \frac{\partial T}{r\,\partial\theta}\right) \cdot \partial z \cdot r\,d\theta$$

$$= \frac{k}{r}\,\frac{dr \cdot r\,d\phi \cdot r\,d\theta}{r}\,\frac{\partial}{\partial\theta}\left(\sin\theta \cdot \frac{\partial T}{\partial\theta}\right) \cdot \partial z$$

$$dQ'_{\theta} = K.\, dr .\, r d\theta\, r\sin\theta d\phi .\, \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left[\sin\theta . \frac{\partial T}{\partial\theta}\right] \partial z$$

Heat flow in r-Direction.

$$Q'_r = -K\,(r d\theta .\, r\sin\theta . d\phi)\, \frac{\partial T}{\partial r} . \partial z$$

$$Q'_{r+dr} = Q'_r + \frac{\partial}{\partial r}(Q'_r) . dr$$

$$dQ'_r = Q'_r - Q'_{r+dr}$$

$$= -\frac{\partial}{\partial r}\left(-K\, r d\theta .\, r\sin\theta\, d\phi . \frac{\partial T}{\partial r} . \partial z\right) . dr$$

$$= K\, d\theta\, \sin\theta\, d\phi\, dr\, \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) \partial z$$

B. Heat generated with in element

$$q'_g = q_g \, (r \, dr \, d\theta \; r \sin\theta \, d\phi) \, \partial\tau$$

C. Heat Stored in element

$$m c \frac{\partial T}{\partial \tau} . \partial\tau$$

$$\rho \, (dr . r d\theta \; r \sin\theta d\phi) \, c \frac{\partial T}{\partial \tau} . \partial\tau$$

$$\underline{k\, r\, d\theta\, dr\, r\sin\theta\, d\phi}\left[\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial T}{\partial r}\right)\right]\underline{\partial\tau}$$

$$+\, q_g\, \underline{r\,dr\,d\theta\, r\sin\theta\, d\phi\,.\,\partial\tau} = \rho\, \underline{r\,dr\,d\theta\, r\sin\theta\, d\phi}\,.\,c\,\frac{\partial T}{\partial\tau}\cdot\partial\tau$$

$$\boxed{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial T}{\partial r}\right)+\frac{q_g}{k}=\frac{\rho . c}{k}\frac{\partial T}{\partial\tau}=\frac{1}{\alpha}\cdot\frac{\partial T}{\partial\tau}}$$