Heat Conduction through Hollow Sphere

r_1 = inner radius

r_2 = outer "

T_1 & T_2 = Temp. at Diff. surface

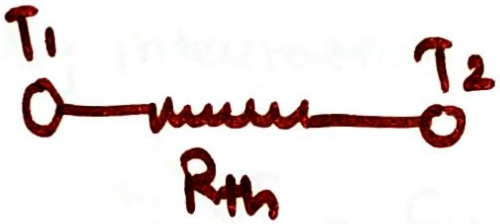
K = Thermal Conductivity

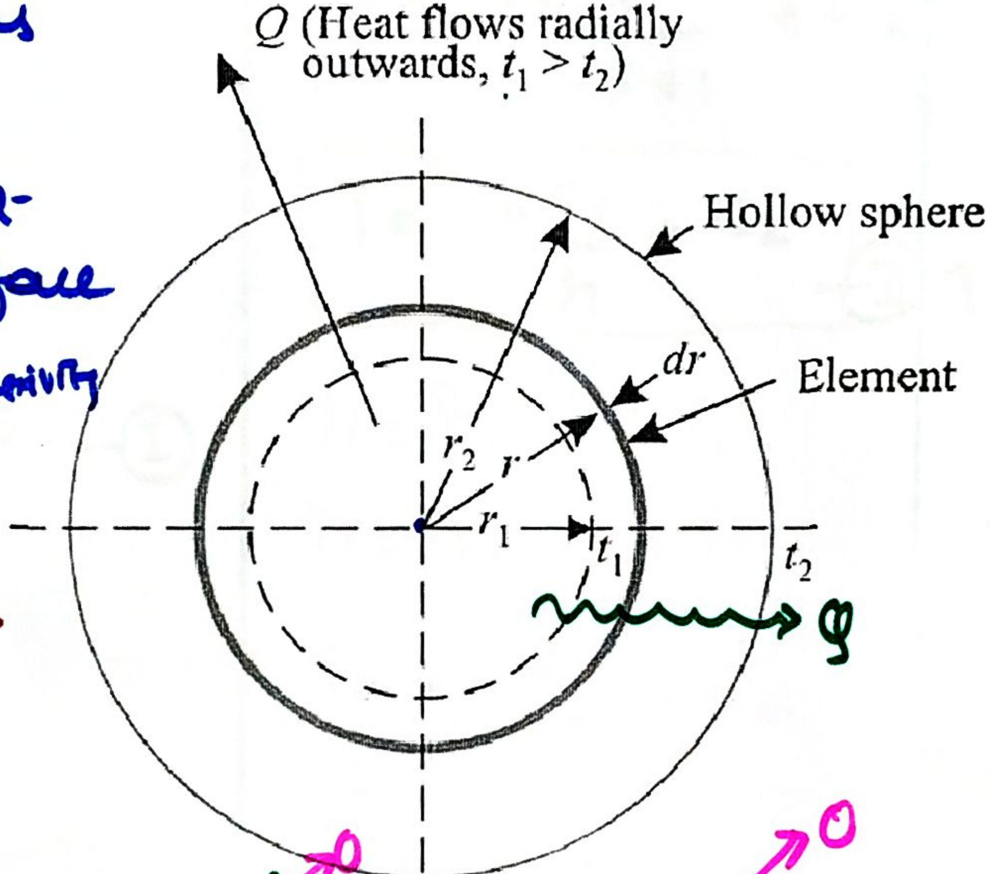
1) Steady State

$\frac{\partial T}{\partial \tau} = 0$

2) Uni Directional θ, ϕ

3) $q_g = 0$





$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}}_{0} + \underbrace{\cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)}}_{0} + \underbrace{\cancel{\frac{q_g}{k}}}_{0} = \underbrace{\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}}_{0}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

$$\underbrace{\frac{1}{r^2}}_{①}\underbrace{\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)}_{②}=0$$

$X \cdot Y = 0$

$\frac{1}{r^2} \neq 0 \quad Y = 0$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0 \quad \text{— I}$$

By integration.

$$r^2\frac{\partial T}{\partial r}=C_1$$

$$\frac{\partial T}{\partial r}=\frac{C_1}{r^2}$$

$$\frac{\partial T}{\partial r}=C_1 \cdot r^{-2}$$

$$\frac{\partial T}{\partial r}=C_1 r^{-2}$$

$$T=C_1\frac{r^{-2+1}}{-2+1}+C_2$$

$$\boxed{T=-\frac{C_1}{r}+C_2} \quad \text{— II}$$

$r=r_1 \quad T=T_1$

$r=r_2 \quad T=T_2$

$$T_1=-\frac{C_1}{r_1}+C_2$$

$$T_2=-\frac{C_1}{r_2}+C_2$$

$$C_1=\frac{(T_1-T_2)r_1r_2}{r_1-r_2}$$

$$C_2=T_1+\frac{(T_1-T_2)r_1r_2}{r_1(r_1-r_2)}$$

$$T=-\frac{(T_1-T_2)r_1r_2}{r(r_1-r_2)}+T_1+\frac{(T_1-T_2)r_1r_2}{r_1(r_1-r_2)}$$

$$T=T_1+\frac{(T_1-T_2)r_1r_2}{(r_1-r_2)}\left[\frac{1}{r_1}-\frac{1}{r}\right]$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{r_2}{r}\left[\frac{r - r_1}{r_2 - r_1}\right]$$

$\boxed{\text{hyperbolas}}$

→ heat Transfer Φ

$$\Phi = -K \underline{A} \frac{dT}{dr}$$

$$A = 4\pi r^2$$

$$T = T_1 + \frac{(T_1 - T_2) r_1 r_2}{(r_1 - r_2)} \cdot \left[\frac{1}{r_1} - \frac{1}{r}\right]$$

$$\frac{dT}{dr} = 0 + \frac{(T_1 - T_2) r_1 r_2}{(r_1 - r_2)} \left[0 - \left(-\frac{1}{r^2}\right)\right]$$

$$\frac{dT}{dr} = \frac{(T_1 - T_2) r_1 r_2}{(r_1 - r_2)} \cdot \frac{1}{r^2}$$

$$\Phi = -K\, 4\pi r^2 \frac{(T_1 - T_2) r_1 r_2}{(r_1 - r_2)} \cdot \frac{1}{r^2}$$

$$\Phi = \frac{4\pi K (T_1 - T_2) r_1 r_2}{(r_2 - r_1)}$$

$$\Phi = \frac{(T_1 - T_2)}{\frac{1}{4\pi K} \frac{(r_2 - r_1)}{r_1 r_2}} \qquad \frac{\Delta T}{R_{th}}.$$

$$R_{th} = \frac{1}{4\pi K} \frac{(r_2 - r_1)}{r_1 r_2}$$

$$R_{th} = \frac{x}{AK}$$

$$R_{th} = \frac{1}{2\pi K l} \ln(r_2/r_1)$$