Critical Thickness of Insulation

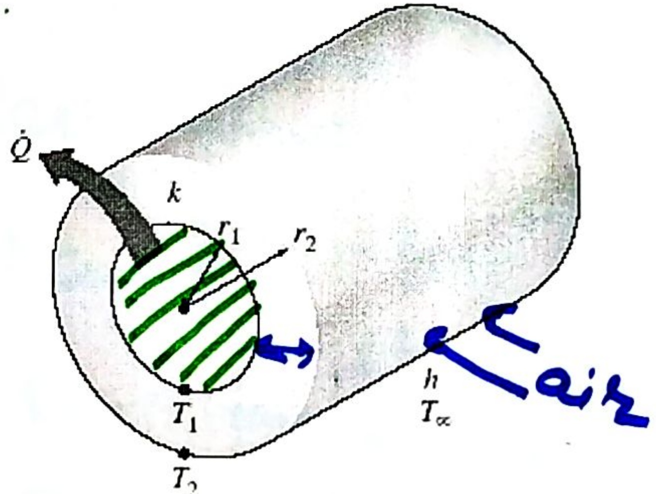
φ Supply to system

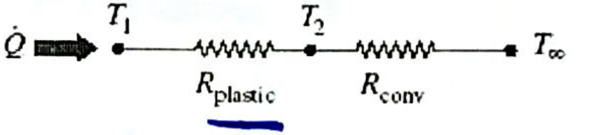
1) Steam Pipe

2) Suction Pipe of Refrigeration System.

Cylinder, Sphere

1) Cylinder





$$R_{total} = R_{\underset{insula}{th}} + R_{\underline{\text{Convection}}\ \text{Outerlayer}}$$

$$= \frac{1}{2\pi k l} \ln r_2/r_1 + \frac{1}{h_{air}} \cdot \frac{1}{2\pi r_2 l}$$

thickness ↑ $\underline{r_2}$ ↑ variation in both Resistance

$$R_{th\,total} = \frac{1}{2\pi k l} \ln r_2/r_1 + \frac{1}{2\pi r_2 l} \cdot \frac{1}{h_{air}}$$

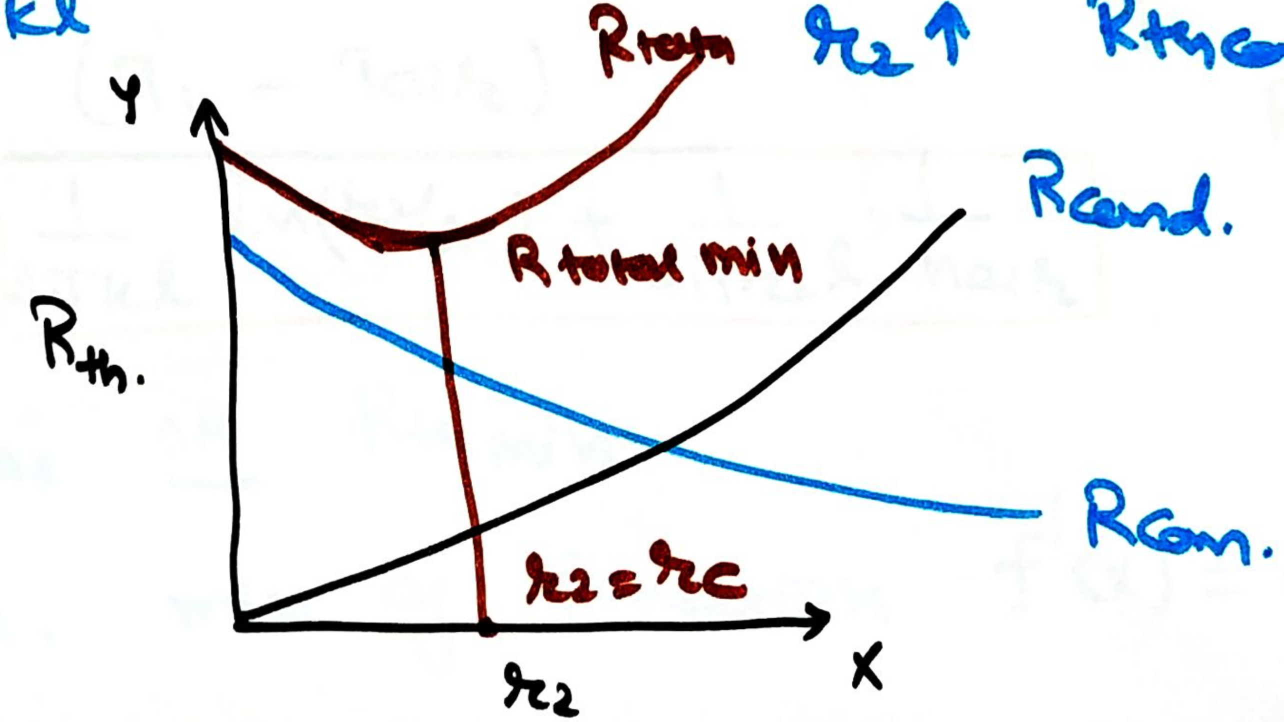
now As $r_2 \uparrow$

$$\frac{1}{2\pi \underline{r_2} l} \cdot \frac{1}{h_{air}} \downarrow = \text{Convection Resistence}$$

$r_2 \uparrow$ $R_{conv} \downarrow$

$$\frac{1}{2\pi k l} \ln r_2/r_1 = \text{Co\underline{nd}uction Resistence}$$

$r_2 \uparrow$ $R_{th\,cond} \uparrow$

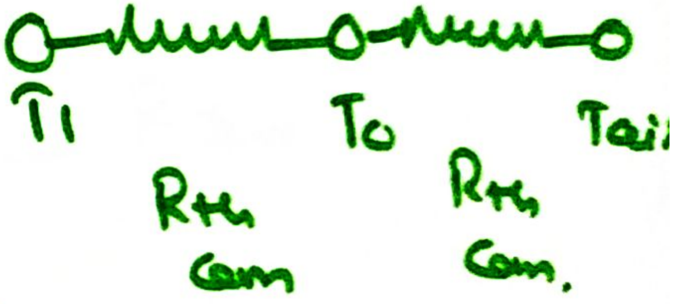


1) critical thickness of insulation for cylinder

$$\varphi = \frac{T_1 - T_{air}}{R_{total}}$$

$$R_{total} = R_{th\,cond} + R_{th\,cond}$$

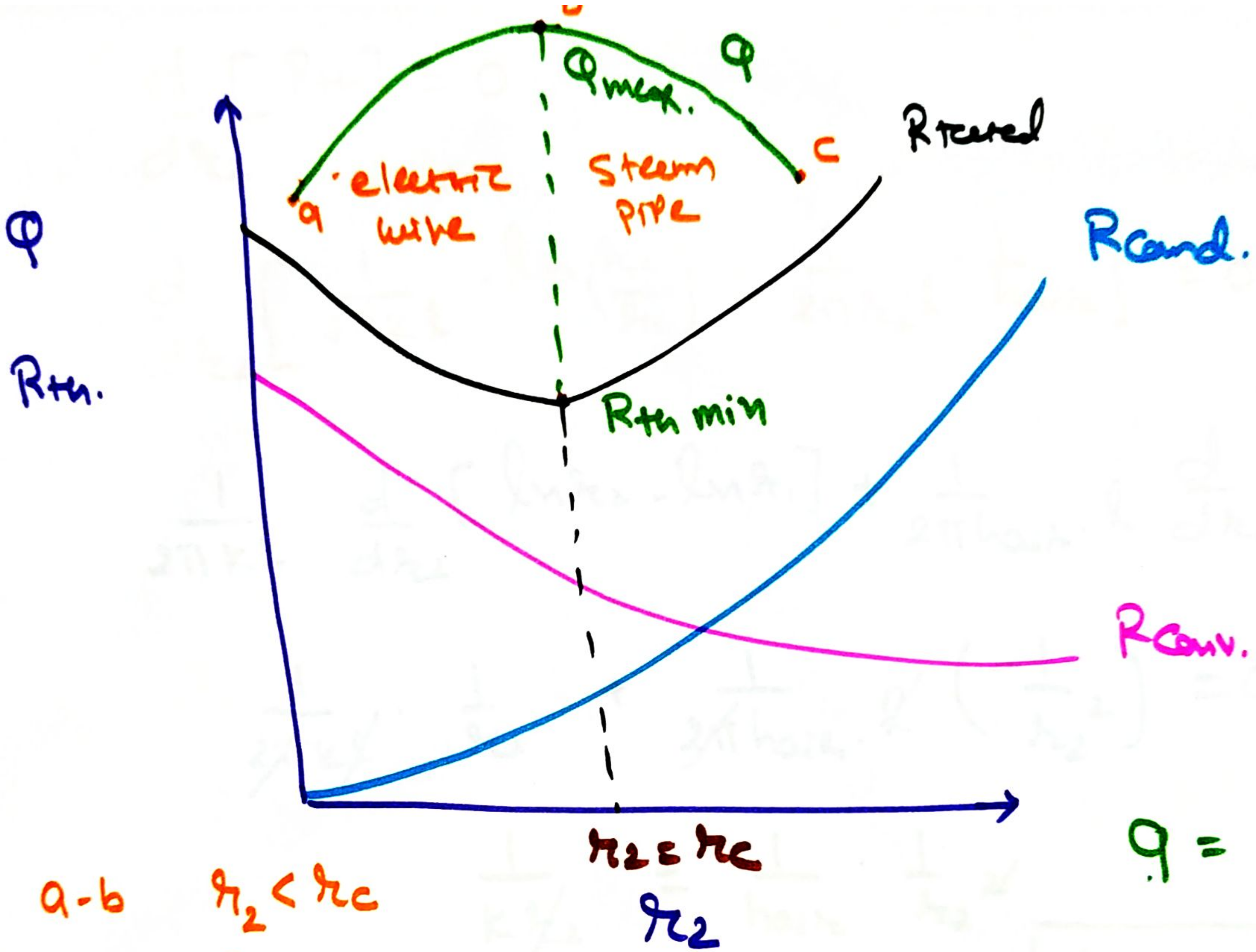
$$R_{total} = \frac{1}{2\pi k l}\ln(r_2/r_1) + \frac{1}{2\pi r_2 l}\cdot\frac{1}{h_{air}}$$



$$\varphi = \frac{(T_1 - T_{air})}{\boxed{\frac{1}{2\pi k l}\ln(r_2/r_1) + \frac{1}{2\pi r_2 l}\cdot\frac{1}{h_{air}}}}$$

φ_{max} **OR** $R_{th\,min}$

max. min of function $f'(x) = 0$



a-b $r_2 < r_c$

b-c $r_2 > r_c$

$$q = \frac{\Delta T}{R_{th}}$$

q_{min} $R_{th\,max}$

q_{max} $R_{th\,min}$

$$\frac{d}{dr_2}[R_{th}] = 0$$

$$\frac{d}{dr_2}\left[\frac{1}{2\pi k l}\cdot \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi r_2 l}\cdot\frac{1}{h_{air}}\right] = 0$$

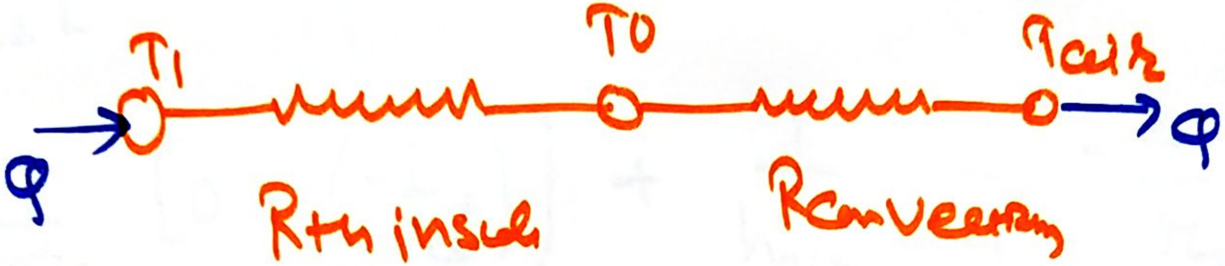
$$\frac{1}{2\pi k l}\,\frac{d}{dr_2}\left[\ln r_2 - \ln r_1\right] + \frac{1}{2\pi h_{air}\cdot l}\,\frac{d}{dr_2}\left(\frac{1}{r_2}\right) = 0$$

$$\frac{1}{\cancel{2\pi} k \cancel{l}}\cdot\frac{1}{r_2} + \frac{1}{\cancel{2\pi} h_{air}\cdot\cancel{l}}\left(-\frac{1}{r_2^2}\right) = 0$$

$$\frac{1}{k\cancel{r_2}} = \frac{1}{h_{air}}\cdot\frac{1}{r_2^{\cancel{2}}}$$

$$r_2 = K/h_{air} = \boxed{\frac{K_{insulation}}{h_{air}}}$$

2) Critical thickness of insulation for sphere



$$\frac{1}{4\pi K}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad \frac{1}{h_{air}}\cdot\frac{1}{4\pi r_2^2}$$

$$\varphi = \frac{(T_1 - T_{air})}{R_{total}} = \frac{(T_1 - T_{air})}{\frac{1}{4\pi K}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) + \frac{1}{h_{air}}\cdot\frac{1}{4\pi r_2^2}}$$

φ_{max}. OR $R_{th\,min}$

$$\frac{d}{dr_2}\left[R_{th}\right] = 0$$

$$\frac{d}{dr_2}\left[\frac{1}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\frac{1}{h_{air}}\cdot\frac{1}{4\pi r_2^2}\right]=0$$

$$\frac{1}{\cancel{4}\pi k}\left[0-\left(-\frac{1}{r_2^2}\right)\right]+\frac{1}{h_{air}}\cdot\frac{1}{\cancel{4\pi}}\cdot\frac{-2}{r_2^3}=0$$

$$\frac{1}{k}\cdot\frac{1}{\cancel{r_2^2}}+\frac{1}{h_{air}}\cdot\frac{-2}{r_2^{\cancel{3}}}=0$$

$$\frac{1}{k}=\frac{2}{h_{air}}\cdot\frac{1}{r_2}$$

$$r_2=\frac{2k}{h_{air}}$$

$$\boxed{r_2=\frac{2\,k_{insulation}}{h_{air}}}$$