$\theta = c_1 e^{mx} + c_2 e^{-mx}$

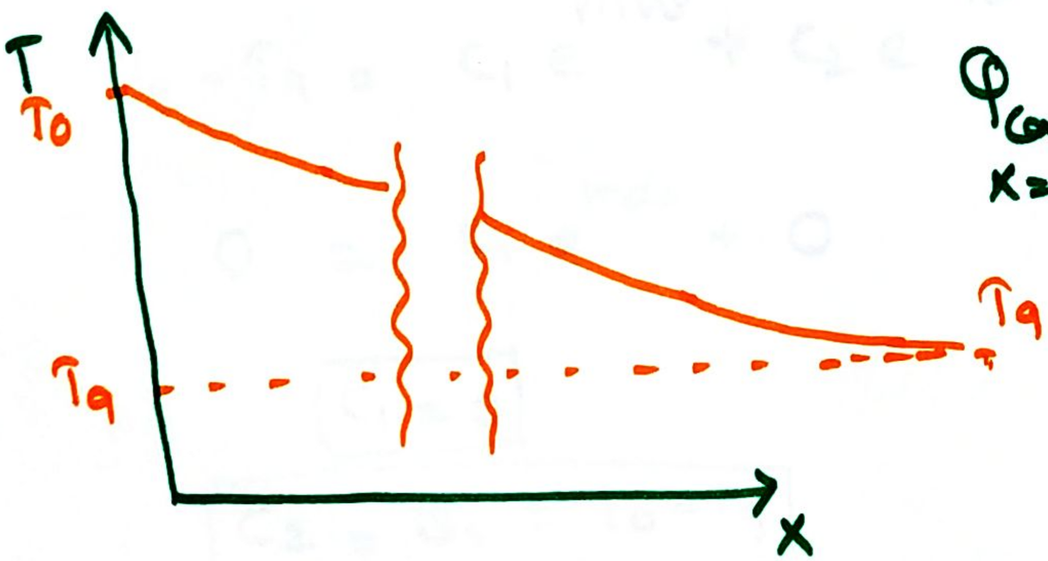
Case - I fin is very long (infiniteely long fin)

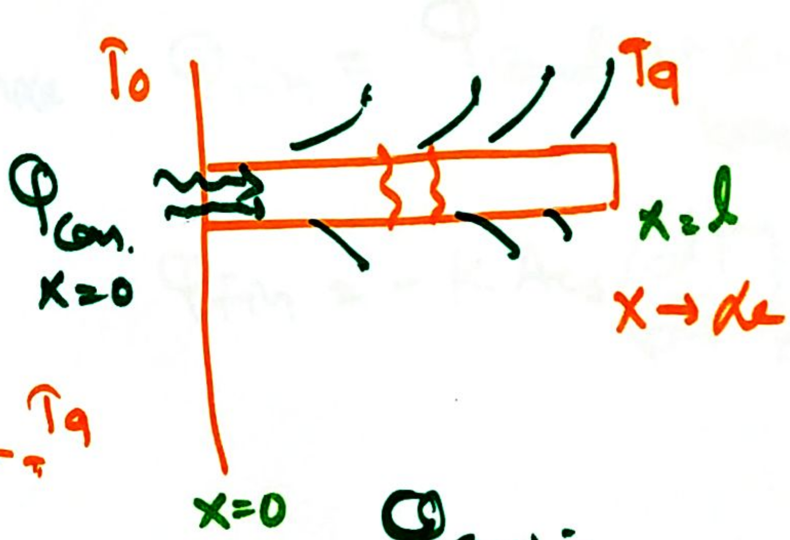
Case - II fin with insulated tip

Case - III fin with convection heat loss from fin tip (Short fin)

## Case - I

fin is very long




$Q_{conv} =$

1) $X=0 \quad T=T_0 \quad \theta=\theta_0$

2) $X=\alpha=l \quad T=T_a \quad \theta_\alpha=0$

$$\theta = c_1 e^{mx} + c_2 e^{-mx}$$

(i)

$$T_0 - T_a = c_1 e^{m(0)} + c_2 e^{-m(0)}$$

$$T_0 - T_a = c_1 + c_2$$

$$\theta_0 = c_1 + c_2$$

(ii)

$$T_a - T_a = c_1 e^{m\alpha} + c_2 e^{-m\alpha}$$

$$0 = c_1 e^{m\alpha} + 0$$

$$\boxed{c_1 = 0}$$

$$\boxed{c_2 = \theta_0 = T_0 - T_a}$$

$$\theta = 0 + (T_0 - T_a) e^{-mx}$$

$$T - T_a = (T_0 - T_a) e^{-mx}$$

$$\frac{T - T_a}{T_0 - T_a} = e^{-mx}$$

→ heat Dissipation from fin

$$Q_{fin} = Q_{cond \text{ at } x=0 \text{ base}}$$

$$Q_{fin} = -K A_{cs} \left(\frac{dT}{dx}\right)_{x=0}$$

$$T - T_a = (T_0 - T_a)\, e^{-mx}$$

$$\frac{d}{dx}(T - T_a) = \frac{d}{dx}\left((T_0 - T_a)\, e^{-mx}\right)$$

$$\frac{dT}{dx} - 0 = (T_0 - T_a)\, \frac{d}{dx}(e^{-mx})$$

$$\frac{dT}{dx} = (T_0 - T_a)\,(-m)\, e^{-mx}$$

$$\left(\frac{dT}{dx}\right)_{x=0} = (T_0 - T_a)\,(-m)\, e^{-m(0)}$$

$$= (T_0 - T_a)(-m)$$

$$q_{fin} = -K A_{cs}\left(\frac{dT}{dx}\right)_{x=0}$$

$$q_{fin} = -K A_{cs}(T_0 - T_a)(-m)$$

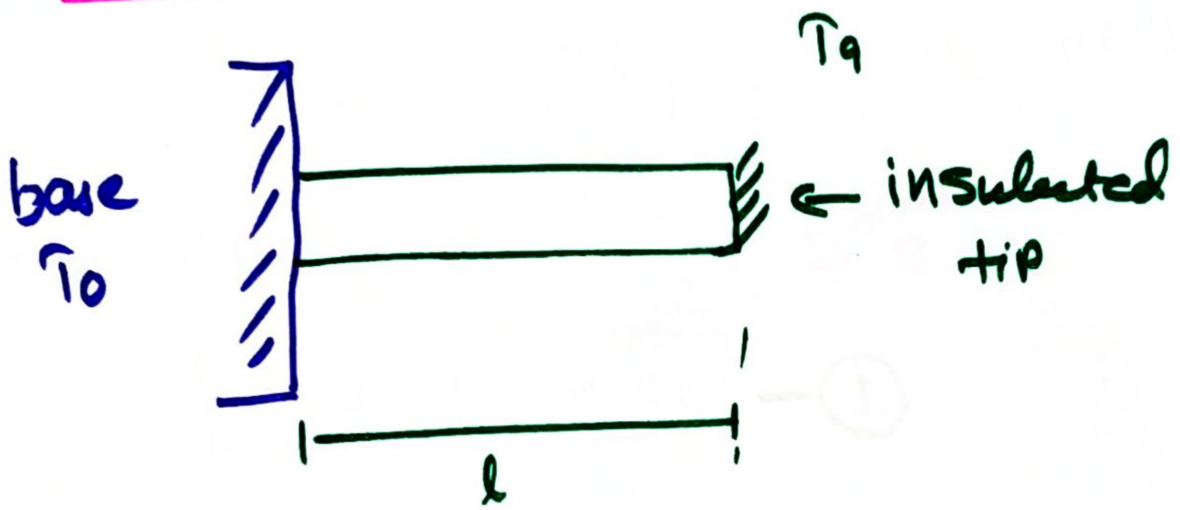
$$= m K A_{cs} (T_0 - T_a)$$

$$m = \sqrt{\frac{P \cdot h}{A_{cs} K}}$$

$$q_{fin} = K A_{cs} \sqrt{\frac{h \cdot P}{A_{cs} K}}\, (T_0 - T_a)$$

$$q_{fin} = \sqrt{h \cdot P \cdot A_{cs} \cdot K}\, (T_0 - T_a)$$

## Case-II Fin with insulated tip.

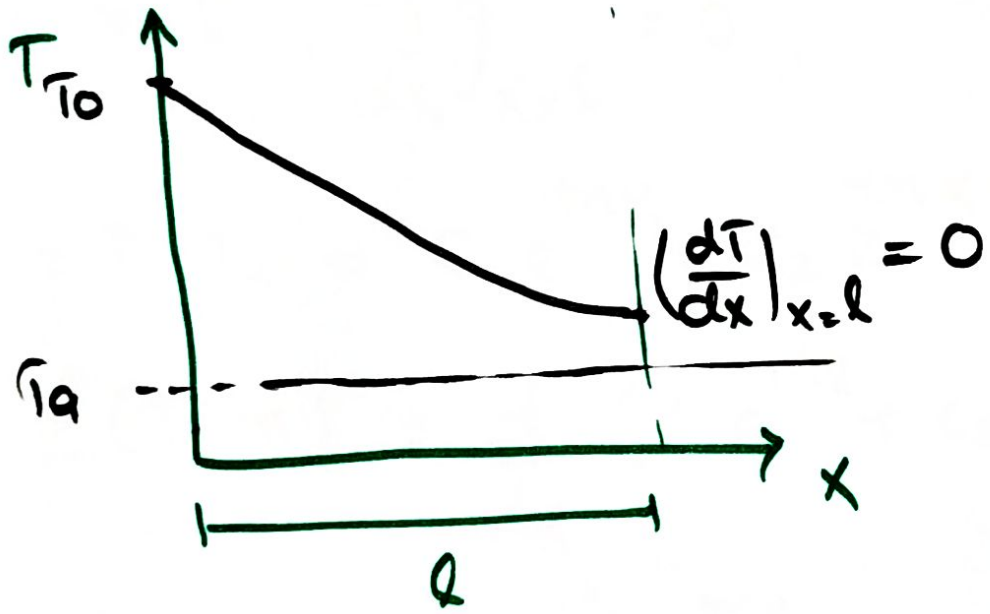



(i) $x = 0$ $T = T_0$

$\theta = \theta_0$

(ii) $x = l$ $\left(\frac{dT}{dx}\right) = 0$

$$\theta = c_1 e^{+mx} + c_2 e^{-mx}$$



(i) $x = 0 \quad \theta = \theta_0 \quad T = T_0$

$$\theta = c_1 e^{mx} + c_2 e^{-mx}$$

$$\theta_0 = c_1 e^{m(0)} + c_2 e^{-m(0)}$$

$$\boxed{\theta_0 = c_1 + c_2} \quad \text{— (I)}$$

(ii) $x = l \quad \left(\frac{dT}{dx}\right)_{x=l} = 0$

$$\theta = T - T_a = c_1 e^{mx} + c_2 e^{-mx}$$

$$\frac{d}{dx}(T - T_a) = \frac{d}{dx}\left(c_1 e^{mx} + c_2 e^{-mx}\right)$$

$$\frac{dT}{dx} - 0 = c_1 m e^{mx} + c_2 - m e^{-mx}$$

$$\boxed{\frac{dT}{dx} = m c_1 e^{mx} + - m c_2 e^{-mx}}$$

$$\left(\frac{dT}{dx}\right)_{x=l} = m c_1 e^{mL} + - m c_2 e^{-mL}$$

$$0 = m c_1 e^{mL} + - m c_2 e^{-mL}$$

Put value of $c_2$ from eq. I

$$c_2 = \theta - c_1$$

$$0 = m c_1 e^{mL} + - m(\theta - c_1) e^{-mL}$$

$$m c_1 e^{mL} - m(\theta_0 - c_1) e^{-mL} = 0$$

$$c_1 e^{mL} - \theta_0 e^{-mL} + c_1 e^{-mL} = 0$$

$$c_1 (e^{mL} + e^{-mL}) - \theta_0 e^{-mL} = 0$$

$$\boxed{c_1 = \frac{\theta_0 e^{-mL}}{e^{mL} + e^{-mL}}}$$

$$c_2 = \theta_0 - c_1$$

$$= \theta_0 - \frac{\theta_0 e^{-mL}}{e^{mL} + e^{-mL}}$$

$$\boxed{c_2 = \frac{\theta_0 e^{mL}}{e^{mL} + e^{-mL}}}$$

$$\theta = c_1 e^{mx} + c_2 e^{-mx}$$

$$\theta = \frac{\theta_0 e^{-mL}}{e^{mL} + e^{-mL}} \cdot e^{mx} + \frac{\theta_0 e^{mL}}{e^{mL} + e^{-mL}} \cdot e^{-mx}$$

$$\frac{\theta}{\theta_0} = \frac{e^{-m(l-x)}}{e^{mL} + e^{-mL}} + \frac{e^{m(l-x)}}{e^{ml} + e^{-ml}}$$

$$\boxed{\cosh x = \frac{e^{x} + e^{-x}}{2}}$$

$$\frac{\theta}{\theta_0} = \frac{\cosh m(l-x)}{\cosh ml}$$

$$Q = -kA_{cs} \cdot -m\,(T_0 - T_a)\tanh ml$$

$$\frac{T - T_a}{T_0 - T_a} = \frac{\cosh m(l-x)}{\cosh ml}$$

$$Q_{fin} = \sqrt{p \cdot h \cdot A_{cs}\, k}\,(T_0 - T_a)\tanh ml$$

The rate of heat Dissipation from fin

$$Q_{fin} = -kA_{cs}\left(\frac{dT}{dx}\right)_{x=0}$$

$$\frac{d}{dx}(T - T_a) = \frac{d}{dx}\left\{(T_0 - T_a) \cdot \frac{\cosh m(l-x)}{\cosh ml}\right\}$$

$$\frac{dT}{dx} - 0 = (T_0 - T_a)\,\frac{-m \sinh ml}{\cosh ml}$$

$$\frac{dT}{dx} = -m\,(T_0 - T_a)\tanh ml$$