Temperature distribution for fins of different configurations

Case	Tip Condition	Temp. Distribution	Fin heat transfer
1	Infinitely long fin θ(L)=0	e^{-mx}	$M\ \theta_0$
2	Adiabatic $(d\theta/dx)_{x=L}=0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\theta_0 \tanh mL$
3	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\theta_o \dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$

$$\theta \equiv T - T_\infty, \qquad m^2 \equiv \frac{hP}{kA_C}$$

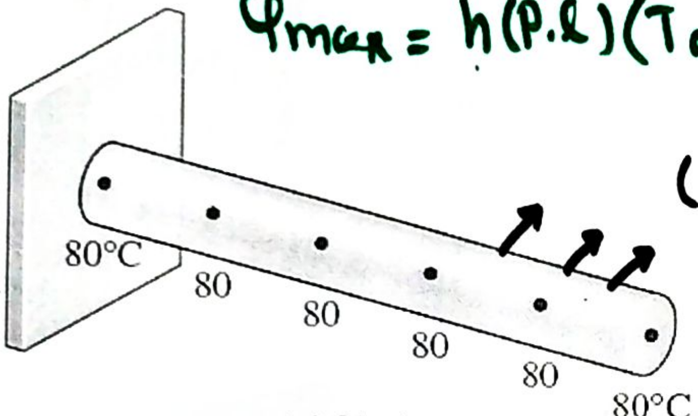
$$\theta_b = \theta(0) = T_b - T_\infty, \qquad M = \sqrt{hPkA_C}\,\theta_b$$

Fin Efficiency

$$\eta_{fin} = \frac{q_{fin}}{q_{max}} = \frac{q_{fin}}{q_{fin(k \to \infty)}}$$

$q = hA\Delta T$

$q_{max} = h(P.l)(T_0 - T_\infty)$

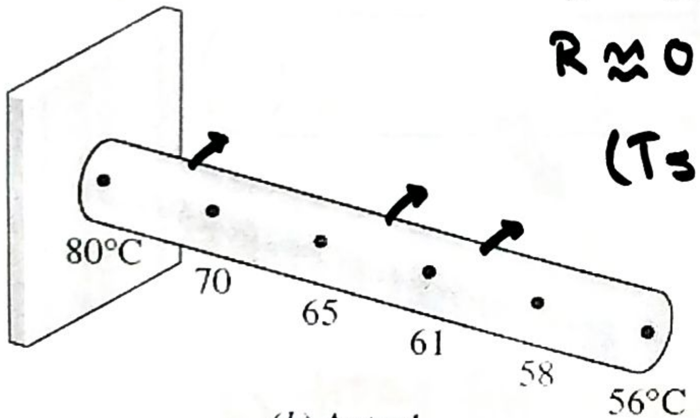


(*a*) Ideal

$(T_s - T_\infty) = c$

$k \to \infty$

$R \approx 0$



(*b*) Actual

$(T_s - T_\infty) \downarrow$

$$\boxed{q_{fin} = \frac{\tanh(ml)}{ml}}$$

a) infinitely long fin ($l \to \infty$)

$$\eta_{fin} = \frac{q_{fin}}{q_{max}} = \frac{\sqrt{hPA_{cs}k}\,(T_0 - T_\infty)}{h\,P.l\,(T_0 - T_\infty)}$$

$$= \sqrt{\frac{hP.A_{cs}k}{h^2 P^2 l^2}}$$

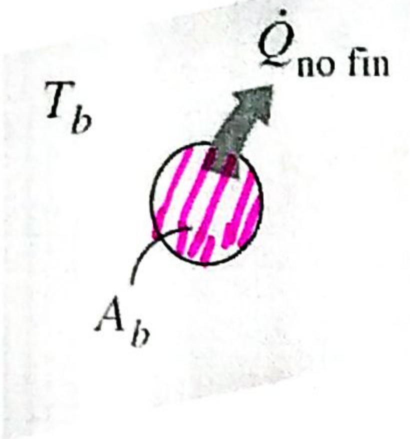
$$\boxed{\eta_{fin} = \frac{1}{ml}}$$

b) tip of fin insulated

$$q_{fin} = \frac{\sqrt{hPA_{cs}k}\,(T_0 - T_\infty)\tanh ml}{h(P.l)(T_0 - T_\infty)}$$

Fin Effectiveness

$$\varepsilon_{fin} = \frac{q_{with\ fin}}{q_{without\ fin}}$$



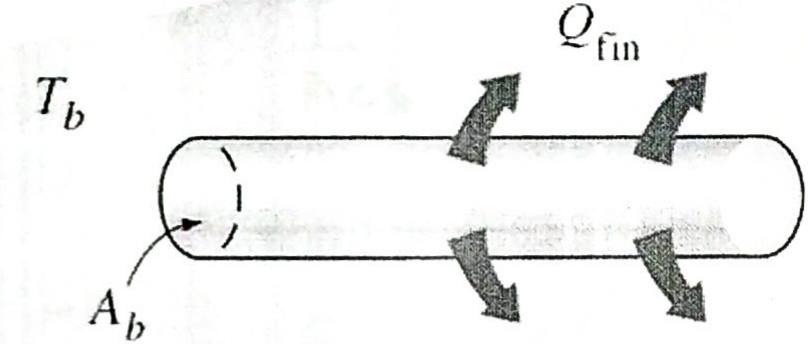
$$q_{without\ fin} = h A_{cs} (T_0 - T_\infty)$$

a) fin with infinitely long

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{without\ fin}}$$

$$\varepsilon_{fin} = \frac{\sqrt{h P A_{cs} k}\,(T_0 - T_\infty)}{h A_{cs} (T_0 - T_\infty)}$$

$$= \sqrt{\frac{P \cdot K}{h \cdot A_{cs}}} \quad \text{— (I)}$$



b) tip of fin insulated

$$\varepsilon_{fin} = \frac{\sqrt{h P A_{cs} k}\,(T_0 - T_\infty) \tanh mL}{h A_{cs} (T_0 - T_\infty)} = \sqrt{\frac{P \cdot K}{h \cdot A_{cs}}} \tanh mL$$

$$\varepsilon_{fin} = \sqrt{\frac{P \cdot K}{h \cdot A_{cs}}}$$

1) K = Thermal Conductivity is high

2) h is small

h → [illegible] }
h → forced convection }

3) $\frac{P}{A_{cs}}$

→

$\varepsilon_{fin} = 1$

$\varepsilon_{fin} \geq 2$ $\quad$ $ml > 5$