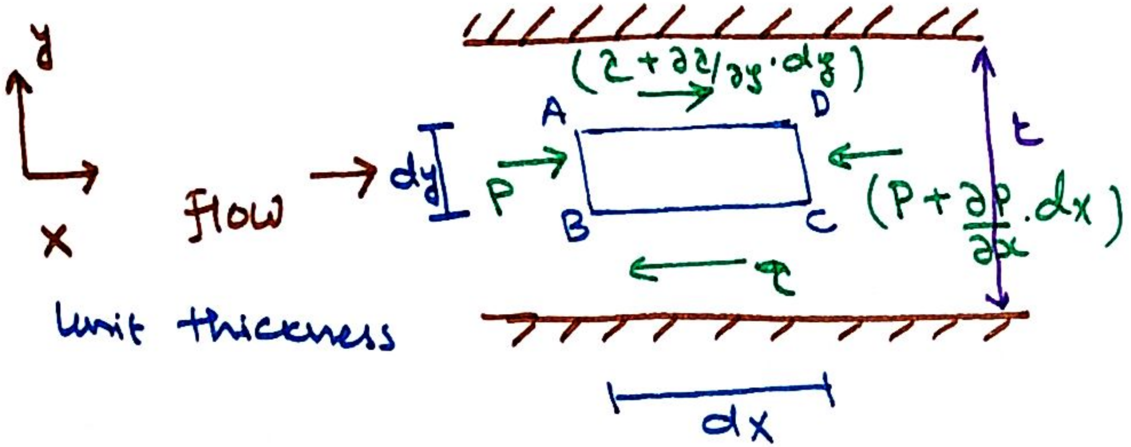
Flow of Viscous Fluid between Two Parallel Fixed Plate



Unit thickness

How for Steady and uniform flow There is no acceleration hence Resultant force is zero

Forces acting on fluid elements

1) Pressure force on AB face

$$P\,(dy \times 1)$$

2) Pressure force on CD face

$$\left(P + \frac{\partial p}{\partial x} \cdot dx\right)(dy \times 1)$$

3) Shear force on BC face

$$\tau \times (dx \times 1)$$

4) Shear force on AD face

$$\left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right)(dx \times 1)$$

$$P\,dy \times 1 - \left(P + \frac{\partial p}{\partial x} dx\right)(dy \times 1) - \tau\, dx \times 1 + \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right)(dx \times 1) = 0$$

$$\cancel{P\,dy} - \cancel{P\,dy} - \frac{\partial p}{\partial x} dx\,dy - \cancel{\tau\,dx} + \cancel{\tau\,dx} + \frac{\partial \tau}{\partial y} \cdot dx\,dy = 0$$

$$-\frac{\partial p}{\partial x} \cancel{dx\,dy} + \frac{\partial \tau}{\partial y} \cdot \cancel{dx\,dy} = 0$$

$$\boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}}$$

(a) Velocity Distribution

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

integrating w.r.t y

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot y + C_1$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot \frac{y^2}{2} + C_1 y + C_2$$

boundary

1) $y=0 \quad u=0$

2) $y=t \quad u=0$

$\rightarrow y=0 \quad u=0$

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot 0 + C_1 0 + C_2$$

$$C_2 = 0$$

$\rightarrow y=t \quad u=0$

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot t^2/2 + C_1 t + 0$$

$$C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\cdot t/2$$

$$C_1 = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot t$$

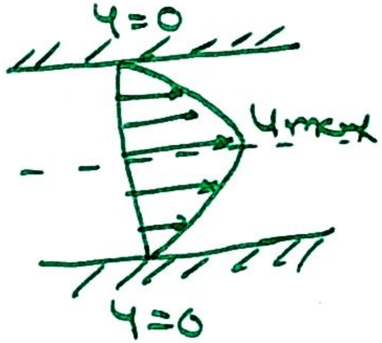
$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot y^2/2 + -\frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot ty$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot y^2 - \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot ty$$

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)$$

$\mu, \frac{\partial p}{\partial x}, t \rightarrow$ constant

$u \rightarrow y^2 \rightarrow$ Parabola



(b) Ratio of max velocity to Avg. Velocity

$y = t/2 \quad u = u_{max}$

$$u = \frac{-1}{2\mu}\frac{\partial p}{\partial x}.(ty - y^2)$$

$y = t/2$

$$u_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t.t/2 - (t/2)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}.\left(t^2/2 - t^2/4\right)$$

$$\boxed{u_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}.t^2}$$

$\bar{u}$ = average velocity

$$dq = u \times A$$
$$= u * (dy \times 1)$$

$$dq = \frac{-1}{2\mu}\frac{\partial p}{\partial x}.(ty - y^2)dy$$

$$q = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}.(ty - y^2)dy$$

$$= \frac{-1}{2\mu}\frac{\partial p}{\partial x}\int_0^t (ty - y^2)dy$$

$$= \frac{-1}{2\mu}\frac{\partial p}{\partial x}\left[t\,y^2/2 - y^3/3\right]_0^t$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}.\left[t^3/2 - t^3/3\right]$$

$$q = -\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^3$$

$$\bar{u} = \frac{q}{A}$$

$$= \frac{-1}{12\mu}\frac{\partial p}{\partial x}.t^{\cancel{3}\,2}/\cancel{t}\times 1$$

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^2$$

$$\frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}.t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^2}$$

$$= 12/8 = 3/2$$

ⓒ Shear stress Distribution.

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\tau = \mu \frac{\partial}{\partial y}\left(-\frac{1}{2\mu}\frac{\partial p}{\partial x}.(ty - y^2)\right)$$

$$\tau = \cancel{\mu}\ -\frac{1}{2\cancel{\mu}}\frac{\partial p}{\partial x}(t - 2y)$$

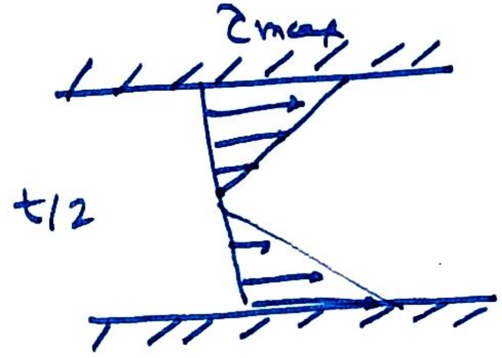
$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}.(t - 2y)$$

τ_{max} $y = 0$

$$\tau_{max} = -\frac{1}{2}\frac{\partial p}{\partial x}.t$$

$\tau = min \rightarrow y = t/2$

$\tau = 0$



ⓓ Pressure Drop

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^2$$

$$\frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

integrating w.r.t. x

$$\int_2^1 \partial p = \int_2^1 -\frac{12\mu\bar{u}}{t^2}dx$$

$$P_1 - P_2 = -\frac{12\mu\bar{u}}{t^2}(x_1 - x_2)$$

$$P_1 - P_2 = -\frac{12\mu\bar{u}}{t^2} \times L$$

h_f = head loss

$$h_f = \frac{P_1 - P_2}{\rho g}$$

$$\boxed{h_f = -\frac{12\mu\bar{u} \times L}{t^2 \rho g}}$$