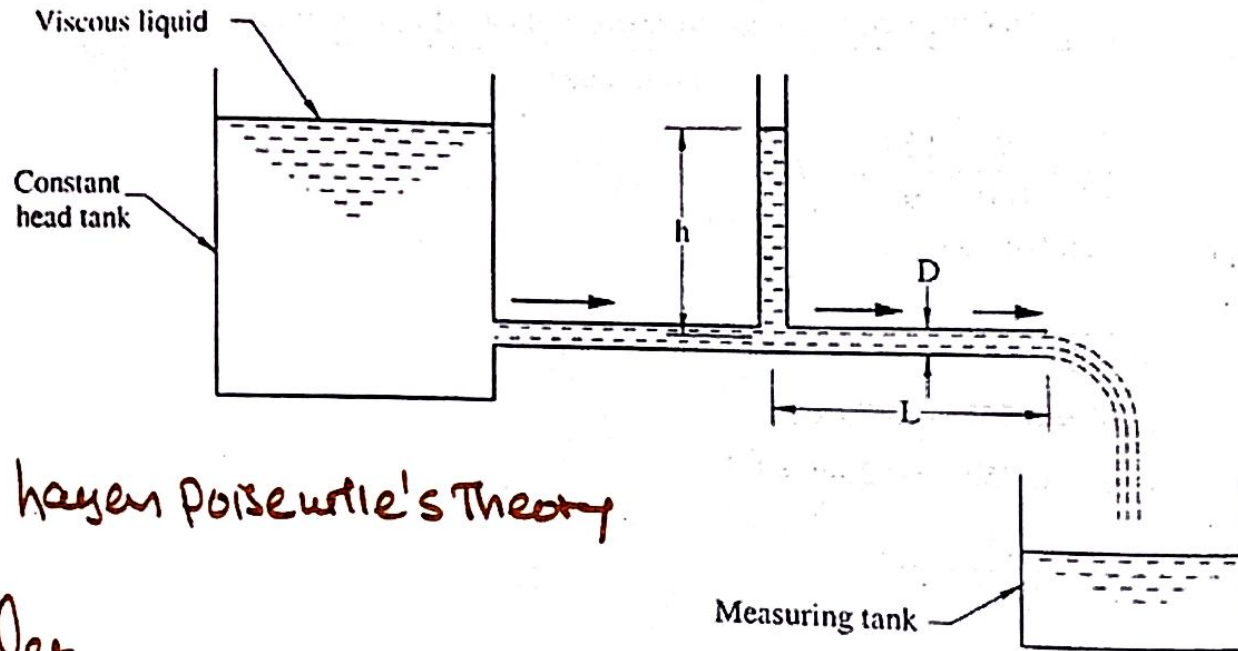


Viscosity Measurement - Capillary Tube Method



→ Hagen Poiseuille's Theory

Let

D = Diameter of Capillary tube

h = Pressure head for length L

μ = Coefficient of viscosity

ρ = Density of fluid

using Hagen Poiseuille's equation

$$h = \frac{32\mu\bar{u}L}{\rho g D^2}$$

\bar{u} = Average Velocity

$$\bar{u} = \frac{Q}{A} = \frac{\Phi}{\pi/4 D^2}$$

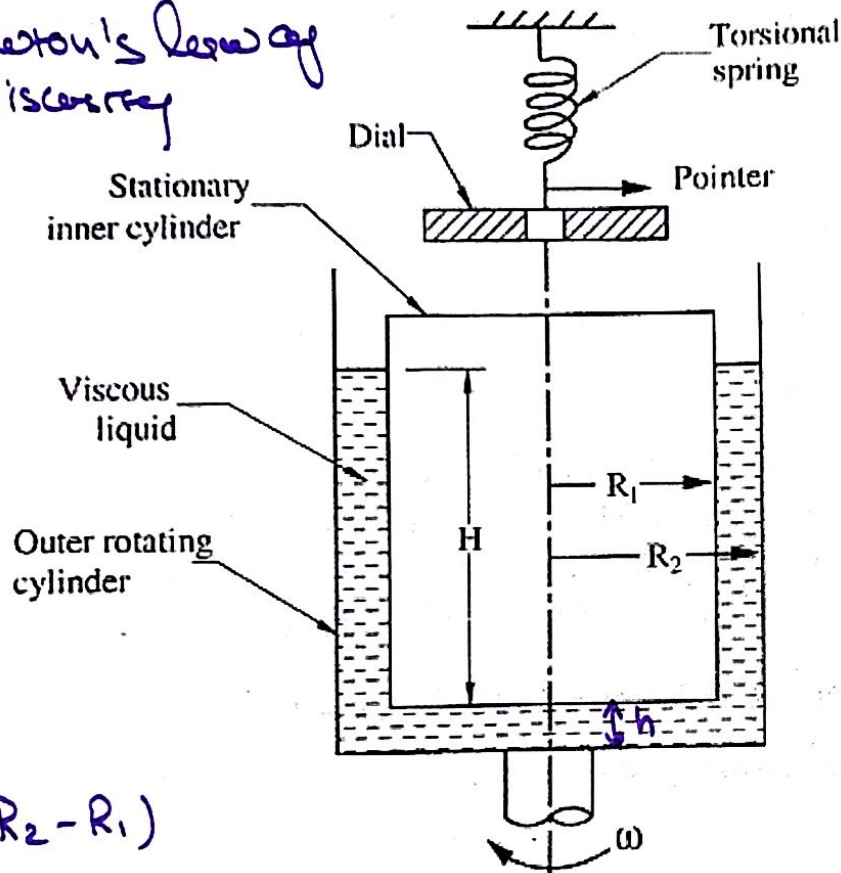
$$h = \frac{32\mu \frac{\Phi}{\pi/4 D^2} \times L}{\rho g D^2}$$

$$h = \frac{128\mu\Phi L}{\pi \rho g D^4}$$

$$\mu = \frac{\pi \rho g D^4}{128\Phi L}$$

Viscosity Measurement - Rotating Cylinder Method

Newton's Law of Viscosity



Velocity of outer cylinder

$$v = R_2 \times \omega$$

inner cylinder = 0

velocity gradient - over the $(R_2 - R_1)$

$$\frac{dv}{dy} = \frac{R_2 \omega - 0}{R_2 - R_1} = \frac{R_2 \omega}{R_2 - R_1}$$

Shear stress

$$\begin{aligned} \tau &= \eta \frac{dv}{dy} \\ &= \eta \times \frac{R_2 \omega}{R_2 - R_1} \end{aligned}$$

Shear force

$$\begin{aligned} F &= \tau \times \text{Surface area} \\ &= \frac{\eta R_2 \omega}{R_2 - R_1} \times 2\pi R_1 H \end{aligned}$$

$$F = \frac{\eta \omega R_2}{R_2 - R_1} \times 2\pi R_1 H$$

$(R_2 - R_1)$

- Let
- ω = angular speed of outer cylinder
 - h = clearance at bottom
 - H = height of liquid in annular space
 - η = coefficient of viscosity
 - T = Torque

Viscosity Measurement - Rotating Cylinder Method

T_1 = Torque on inner cylinder due to shear stress

T_1 = shear force \times Radius

$$= \frac{\mu R_2 \omega}{R_2 - R_1} \times 2\pi R_1 H \times R_1$$

$$T_1 = \frac{2\pi \mu \omega H R_1^2 R_2}{R_2 - R_1}$$

using Theory of fluid bearing

T_2 = Torque applied on inner cylinder

$$T_2 = \frac{\mu}{60t} \times \pi^2 \mu R^4$$

$$R = R_1 \quad t = h$$

$$T_2 = \frac{\mu}{60h} \times \pi^2 \times \mu \times R_1^4$$

T = Total Torque

$$T = T_1 + T_2$$

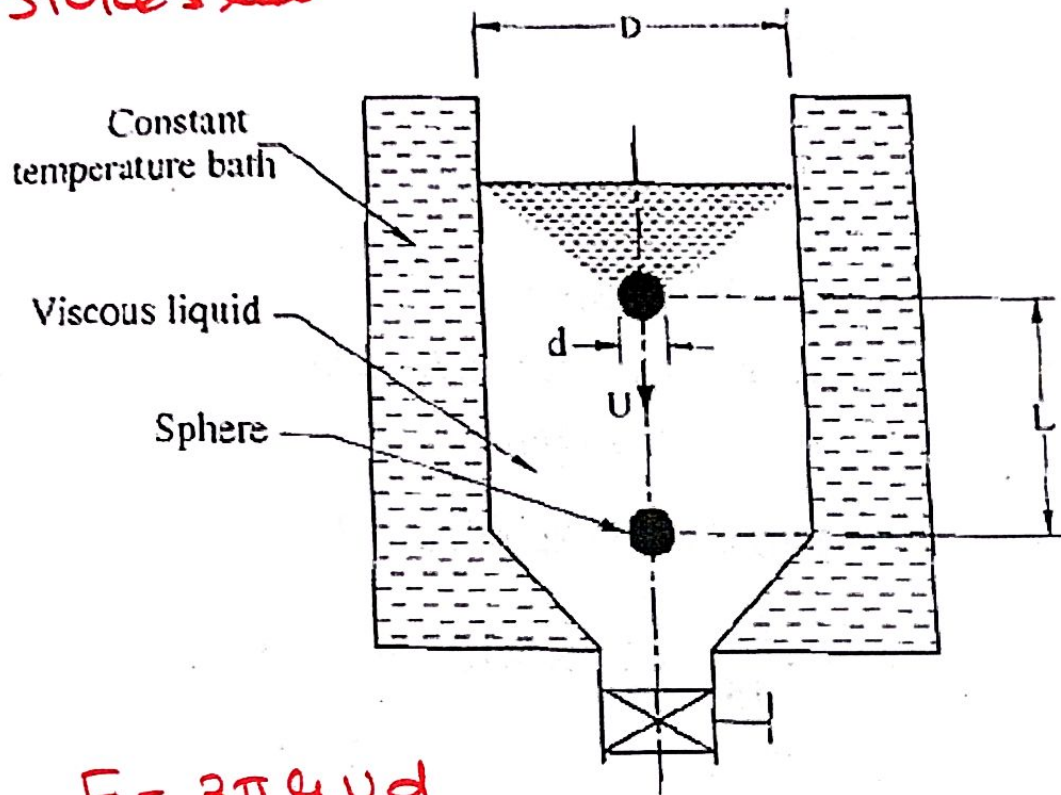
$$T = \frac{2\pi \mu \omega H R_1^2 R_2}{R_2 - R_1} + \frac{\pi \mu \omega R_1^4}{2h}$$

$$T = 2\pi \mu R_1^2 \omega \left[\frac{R_2 H}{R_2 - R_1} + \frac{R_1^2}{4h} \right]$$

$$\mu = \frac{2(R_2 - R_1) h T}{\pi R_1^2 \omega [4Hh R_2 + R_1^2 (R_2 - R_1)]}$$

Viscosity Measurement - Falling Sphere Method

Stokes's Law



$$F = 3\pi\eta Ud$$

d = Dia of sphere

U = velocity of sphere

Let $L =$ Distance Travelled by Spherically viscous liquid

$t =$ time taken by sphere to cover distance L

$\rho_s =$ Density of sphere

$\rho_f =$ Density of fluid

$W =$ weight of sphere

$F_B =$ buoyant force acting on sphere

$$U = \frac{L}{t}$$

$$W = \rho_s V \times g$$

$$= \rho_s \times \frac{\pi}{6} d^3 \times g$$

buoyant force

$$F_B = \text{weight of fluid displaced}$$

$$= \frac{\pi}{6} d^3 \times \rho_f \times g$$

For equilibrium condition

viscous force = weight of sphere - buoyant force

$$3\pi\mu Ud = \rho_s \pi/6 d^3 \times g - \pi/6 d^3 \times \rho_f \times g$$

$$\mu = \frac{\pi/6 d^3 g (\rho_s - \rho_f)}{3\pi Ud}$$

$$\mu = \frac{gd^2 (\rho_s - \rho_f)}{18U}$$