Transient Heat Conduction.

$T = f(x, y, z)$

$T = f(x, y, z, t)$

Application of Transient heat conduction.

1) heating & cooling of metal billets
2) cooling of IC Engine
3) Heat Treatment of metal by quenching

→ The analysis of unsteady state Problem of Heat Transfer

1) Analytical method
2) graphical method
3) Numerical Technique

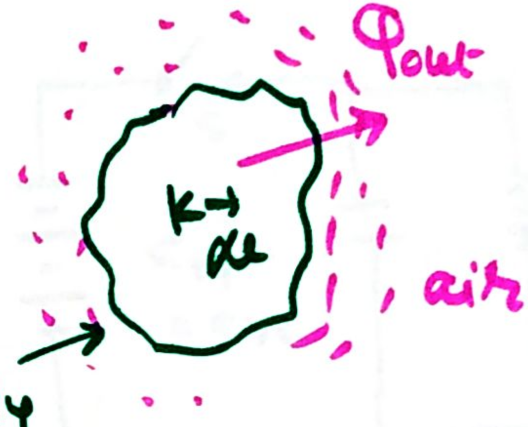
→ a) Periodic variation
b) Non Periodic variation

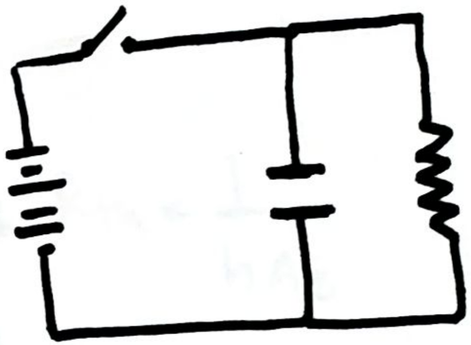
Lumped Parameter Analysis ($K \to \infty$)

Solid K

internal Resistance

$$R_{th} = \frac{x}{AK}$$





air, water

R_{th} = internal Resistance = 0

$\frac{1}{hA}$ = Convective Resistance

0 → 100
1 → 95
2 → 90
3 → 85

entire body → lump

$\tau = 0 \quad T = T_i$

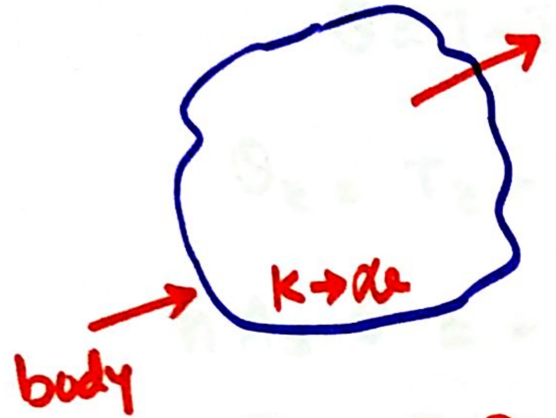
$\tau > 0 \quad T = f(\tau)$

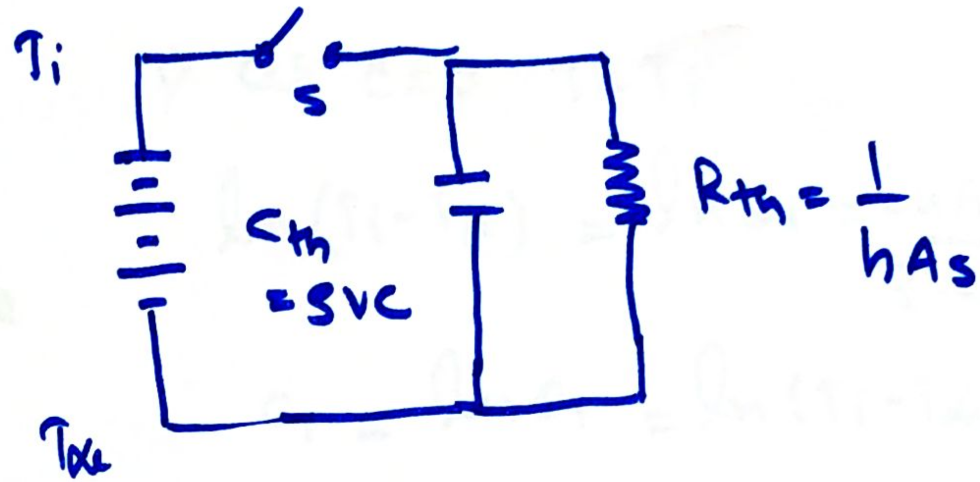
Energy Balance eqn.

Rate of heat flow out the solid = Rate of heat Transfer by Convection at surface

$$Q = \rho V c \frac{dT}{d\tau} = hA(T - T_a)$$

Lumped Parameter Analysis





$\tau = 0 \quad T = T_i$

Energy Balance eqn

The Rate of Decrease in internal energy of body = The Rate of heat flow at boundary of solid

$$- \rho V C \frac{\partial T}{\partial \tau} = h A_s (T - T_\infty)$$

$$hA_s(T-T_\infty) = -\rho V c \frac{\partial T}{\partial z}$$

$$\theta = T - T_\infty$$

$$\theta_z = T_z - T_\infty \qquad \frac{d\theta}{dz} = \frac{dT}{\partial z}$$

$$hA_s\,\theta = -\rho V c \frac{d\theta}{dz}$$

$$\frac{d\theta}{\theta} = -\frac{hA_s}{\rho V c}\,dz$$

integrating both side

$$\int \frac{d\theta}{\theta} = \int -\frac{hA_s}{\rho V c}\,dz$$

$$\ln\theta = -\frac{hA_s}{\rho V c} z + c_1$$

*) Boundary condition.

i) at $z=0 \quad T=T_i$

$$\ln(T_i - T_\infty) = \ln\theta_i = -\frac{hA_s}{\rho V c}(0) + c_1$$

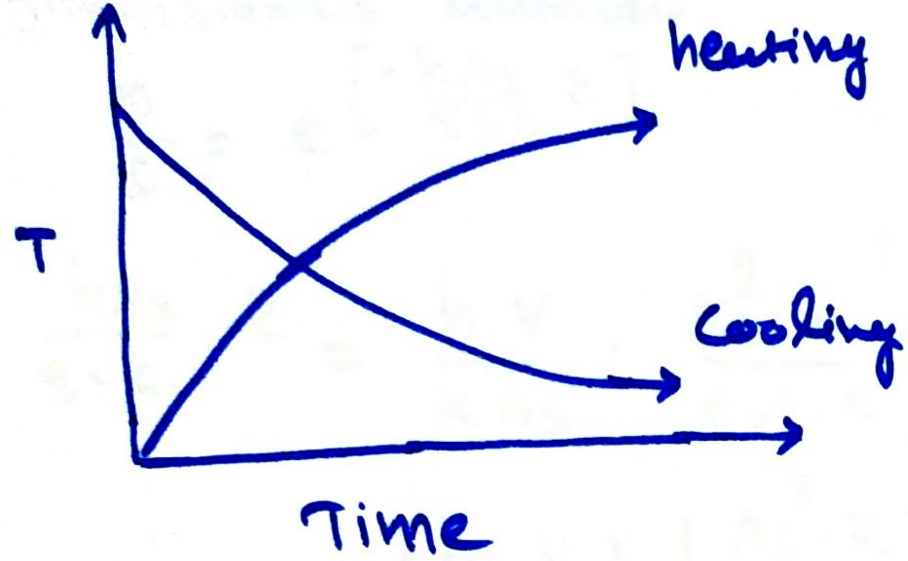
$$c_1 = \ln\theta_i = \ln(T_i - T_\infty)$$

$$\ln\theta = -\frac{hA_s}{\rho V c} z + \ln\theta_i$$

$$\ln\theta - \ln\theta_i = -\frac{hA_s}{\rho V c}\cdot z$$

$$\ln\left(\frac{\theta}{\theta_i}\right) = -\frac{hA_s}{\rho V c} z$$

$$\frac{\theta}{\theta_i} = e^{\left[-\frac{hA_s}{\rho V c} z\right]}$$



$$\frac{\rho V c}{h A_s} = \text{Thermal time constant}$$

$$\tau_{th} = \frac{\rho V c}{h A_s} = \frac{1}{h A_s} \cdot \rho V c$$

$$\tau_{th} = R_{th} \cdot C_{th}.$$

Dimensionless Number

$$\frac{\theta}{\theta_i} = e^{\left[-\frac{hA_s}{\rho V c}\tau\right]}$$

$$\frac{hA_s}{\rho V c}\cdot\tau = \frac{h.V}{KA_s}\cdot\frac{A_s^2.K}{\rho V^2.c}\cdot\tau$$

$$= \left(\frac{h}{K}\cdot\frac{V}{A_s}\right)\left(\frac{A_s^2.K}{\rho V^2 c}\right)\tau$$

$\alpha = \frac{K}{\rho c}$ $\quad L_c$ = characteristic length $= \frac{V}{A_s}$

$$= \left(\frac{h.L_c}{K}\right).\left(\frac{\alpha.\tau}{L_c^2}\right)$$

Biot Number $= Bi = \frac{h.L_c}{K}$

Fourier Number $= F_o = \frac{\alpha\tau}{L_c^2}$

1) Biot Number

Bi = $\frac{\text{Internal Resistance of Solid}}{\text{Convective Resistance at Surface}}$

$$= \frac{L_c}{A_s K} \times \frac{1}{1/hA_s}$$

$$Bi = \frac{h.L_c}{K}$$

⇒ for lump Parameter Analysis

$$\boxed{Bi \leq 0.1}$$

2) Fourier Number

$$F_o = \frac{K A_s\ \Delta T/L_c}{\rho V c\ \Delta T/\tau} = \frac{K.A_s.\tau}{\rho A_s L\ c\ L_c}$$

$$= \frac{K}{\rho c}.\frac{\tau}{L_c^2} = \frac{\alpha\tau}{L_c^2}$$

→ characteristic length.

$$L_c = \frac{V}{A_s}$$

L, B, H

(a) Plate $L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = L/2$

(b) cylinder $L_c = \frac{V}{A_s} = \frac{\pi R^3 L}{2\pi R L} = R/2$

(c) sphere $L_c = \frac{V}{A_s} = \frac{4/3\,\pi r^3}{4\pi r^2} = r/3$

(d) cube $L_c = \frac{l^3}{6l^2} = l/6$