Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i

- It is the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.
- It always exists in the fluid flow problems

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v

➢It is equal to the product of shear stress due to viscosity and surface area of the flow.

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g

➢ It is equal to the product of mass and acceleration due to gravity of the flowing fluid.

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p

➢ It is equal to the product of pressure intensity and cross sectional area of flowing fluid

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p
5. Surface Tension Force, F_s

- It is equal to the product of surface tension and length of surface of the flowing

Types of Forces Acting on Moving Fluid

1. Inertia Force, F_i
2. Viscous Force, F_v
3. Gravity Force, F_g
4. Pressure Force, F_p
5. Surface Tension Force, F_s
6. Elastic Force, F_e

- It is equal to the product of elastic stress and area of the flowing fluid

Dimensionless Numbers

Dimensionless numbers are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force.

1. Reynold's number, R_e = $\dfrac{\text{Inertia Force}}{\text{Viscous Force}} = \dfrac{{}^{\circ}VL}{«}\ or\ \dfrac{{}^{\circ}VD}{«}$

2. Froude's number, F_e = $\sqrt{\dfrac{\text{Ine rtiaForce}}{\text{Gravity Force}}} = \dfrac{V}{\sqrt{Lg}}$

3. Euler's number, E_u = $\sqrt{\dfrac{\text{InertiaForce}}{\text{PressureForce}}} = \dfrac{V}{\sqrt{p / {}^{\circ}}}$

4. Weber number, W_e = $\sqrt{\dfrac{\text{InertiaForce}}{\text{SurfaceTensionForce}}} = \dfrac{V}{\sqrt{\pm / {}^{\circ}L}}$

5. Mach's number, M = $\sqrt{\dfrac{\text{Inertia Force}}{\text{Elastic Force}}} = \dfrac{V}{C}$

R V
F g
W S
E p
M e

Reynold's Number (R_e). It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

Inertia force (F_i) $= \text{Mass} \times \text{Acceleration of flowing fluid}$

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho A V^2$$

Viscous force (F_v) $= \text{Shear stress} \times \text{Area}$ $\left\{\because \ \tau = \mu \dfrac{du}{dy} \ \therefore \ \text{Force} = \tau \times \text{Area}\right\}$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \,.\, \frac{V}{L} \times A \qquad \left\{\because \ \frac{du}{dy} = \frac{V}{L}\right\}$$

By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \,.\, \dfrac{V}{L} \times A} = \frac{\rho V L}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \qquad \left\{\because \ \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}.$$

Froude's Number (F_e). The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

where F_i from equation (12.11) $= \rho AV^2$

and F_g = Force due to gravity

= Mass × Acceleration due to gravity

$= \rho \times \text{Volume} \times g = \rho \times L^3 \times g$ $\{\because \text{ Volume} = L^3\}$

$= \rho \times L^2 \times L \times g = \rho \times A \times L \times g$ $\{\because L^2 = A = \text{Area}\}$

$$\therefore \quad F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho AV^2}{\rho ALg}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

Euler's Number (E_u). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Mathematically, it is expressed as

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

where F_p = Intensity of pressure × Area $= p \times A$

and $F_i = \rho AV^2$

$$\therefore \quad E_u = \sqrt{\frac{\rho AV^2}{p \times A}} = \sqrt{\frac{V^2}{p/\rho}} = \frac{V}{\sqrt{p/\rho}}$$

Weber's Number (W_e). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Weber's Number, $$W_e = \sqrt{\frac{F_i}{F_s}}$$

where F_i = Inertia force = $\rho A V^2$
and F_s = Surface tension force
= Surface tension per unit length × Length = $\sigma \times L$

$$\therefore \quad W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}} \qquad \{\because \; A = L^2\}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}} = \sqrt{\frac{V^2}{\sigma / \rho L}} = \frac{V}{\sqrt{\sigma / \rho L}}.$$

Mach's Number (M). Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

where $F_i = \rho A V^2$
and F_e = Elastic force = Elastic stress × Area
$= K \times A = K \times L^2$ $\qquad \{\because \; K = \text{Elastic stress}\}$

$$\therefore \quad M = \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K / \rho}} = \frac{V}{\sqrt{K / \rho}}$$

But $$\sqrt{\frac{K}{\rho}} = C = \text{Velocity of sound in the fluid}$$

$$\therefore \quad M = \frac{V}{C}.$$