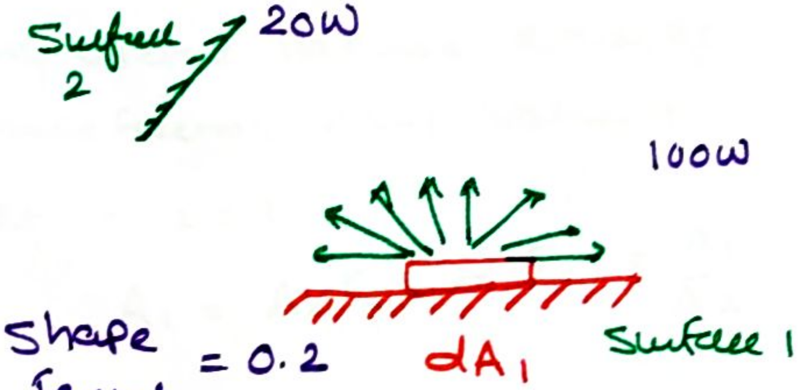
Properties of Shape Factor

Shape Factor: it is defined as the fraction of radiant energy that is diffused from one surface element and strikes to other surface directly with no intervening reflection.



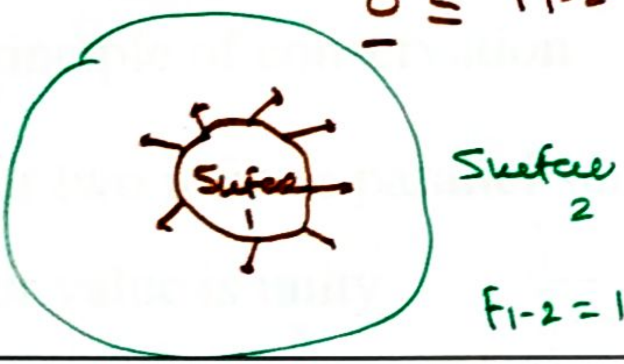
Shape factor $= 0.2$
$= 20\%$

$$\phi_{1-2} = F_{1-2}\,\sigma A_1 (T_1^4 - T_2^4)$$
$$= F_{2-1}\,\sigma A_2 (T_1^4 - T_2^4)$$

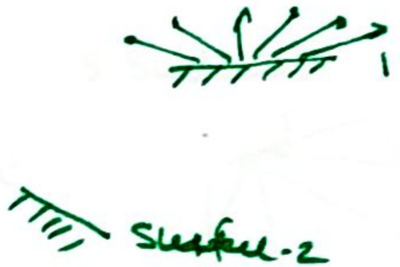
F_{12} & F_{2-1} shape factor

value of shape factor is lies in the Range of 0 to 1

$$0 \le F_{1-2} \le 1$$



$F_{1-2} = 1$



$F_{1-2} = 0$

Properties of Shape Factor

1. The shape factor is purely function of geometry parameter and it is also known as geometry factor

2. Reciprocity Theorem

$$A_1 F_{1-2} = A_2 F_{2-1}$$

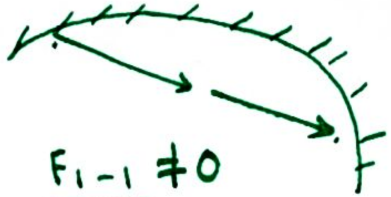
very useful in case either of shape factor have value 1

let $F_{1-2} = 1$

$$A_1 = A_2 F_{21} \Rightarrow F_{21} = \frac{A_1}{A_2}$$

3. Flat and convex surface shape factor with respect to itself is zero

4. For concave surface, the shape factor with respect to itself is not equal to zero



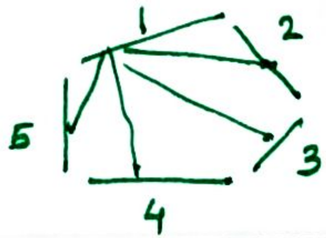
$F_{1-1} \neq 0$

$F_{1-1} = 0$

$F_{1-1} = 0$

5. Principle of conservation

5 surface

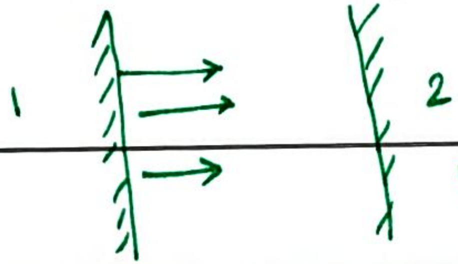


$$F_{1-1} + F_{1-2} + F_{1-3} + F_{1-4} + F_{1-5} = 1$$

$$F_{2-1} + F_{2-2} + F_{2-3} + F_{2-4} + F_{2-5} = 1$$

$$F_{5-1} + F_{52} \text{ -- } + F_{55} = 1$$

6. For two infinite parallel surface shape factor value is unity



$F_{1-2} = F_{2-1} = 1$

Heat Exchange between Non-Black Surface

→ Heat exchange betn Two black surface

$$\Phi_{1-2} = F_{12} A_1 \sigma (T_1^4 - T_2^4)$$

$$= F_{2-1} A_2 \sigma (T_1^4 - T_2^4)$$

→ All incident Radiation because $\alpha = 1$

$\alpha, \varepsilon, \rightarrow \lambda,$ Direction

$\alpha = \alpha_\lambda = C, \quad \varepsilon = \varepsilon_\lambda = C$

→ gray surface

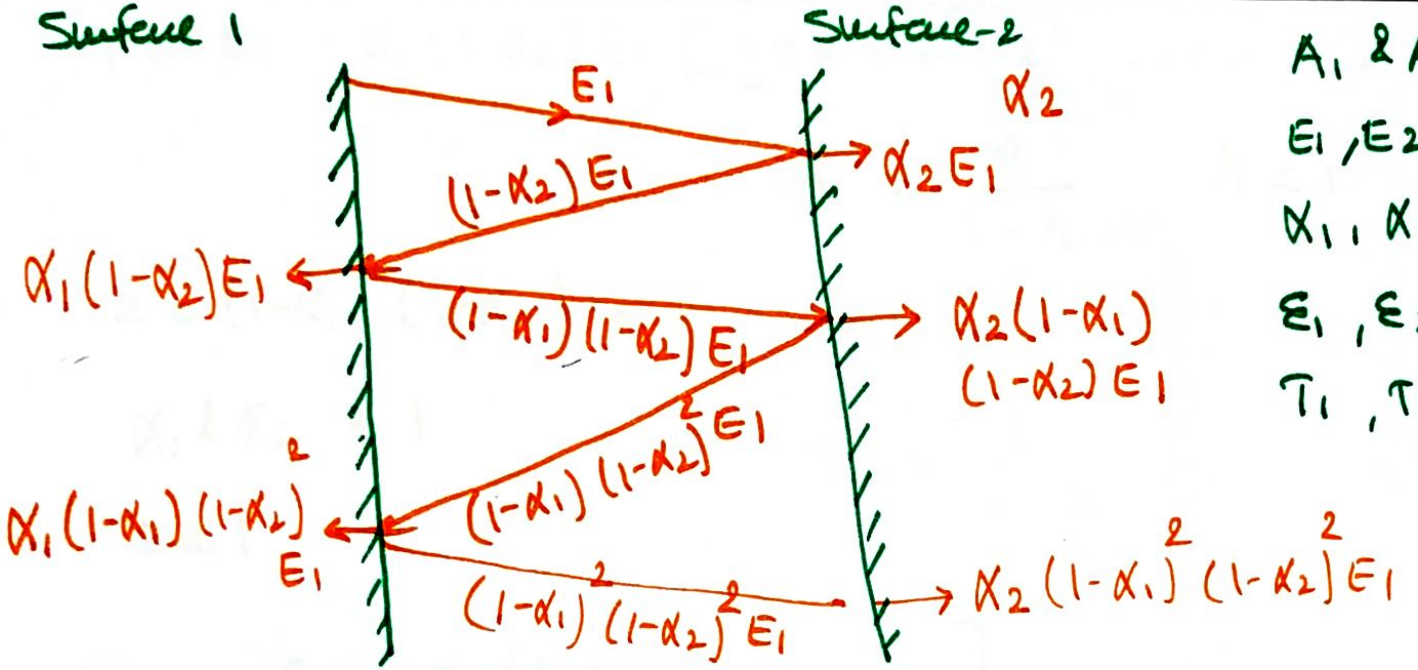
→ Opaque $\tau = 0 \quad \alpha + \delta = 1$

Assumption

1) Two Parallel surface $F_{12} = F_{21} = 1$

2) $\varepsilon, \alpha = C$

3) Non Absorbing material (medium)



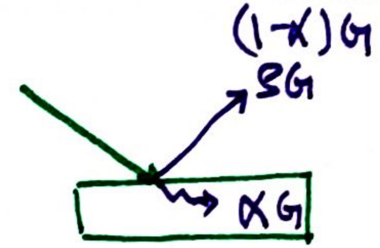
A_1 & A_2

E_1, E_2

α_1, α_2

$\varepsilon_1, \varepsilon_2,$

T_1, T_2



$\tau = 0$

$\alpha + \rho + \tau = 1$

$\alpha + \rho = 1$

$\rho = 1 - \alpha$

The amount of Energy from Surface 1 to 2

$$Q_1 = E_1 - \left[\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \ldots\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \ldots\right]$$

let take $z = (1-\alpha_1)(1-\alpha_2)$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + z + z^2 + z^3 + \ldots\right]$$

$$\Phi_1 = E_1 - \alpha_1(1-\alpha_2)E_1[1 + z + z^2 + z^3 + \dots\dots]$$

$$S_n = \frac{a}{1-r} \qquad r < 1$$

$$z = (1-\alpha_1)(1-\alpha_2) < 1$$

α_1 & $\alpha_2 < 1$

$$S_n = \frac{1}{1-z}$$

$z < 1$

$$\Phi_1 = E_1 - \alpha_1(1-\alpha_2)E_1\left[\frac{1}{1-z}\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[\frac{1}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

$\alpha = \varepsilon \qquad \alpha_1 = \varepsilon_1 \qquad \alpha_2 = \varepsilon_2$

$$= E_1\left[1 - \frac{\varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$= E_1\left[\frac{1-(1-\varepsilon_1)(1-\varepsilon_2) - \varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$\varphi_1 = E_1 \left[\frac{1 - (1-\varepsilon_1)(1-\varepsilon_2) - \varepsilon_1 (1-\varepsilon_2)}{1 - (1-\varepsilon_1)(1-\varepsilon_2)} \right]$$

$$= E_1 \left[\frac{\cancel{1} - \cancel{1} + \varepsilon_2 + \cancel{\varepsilon_1} - \cancel{\varepsilon_1 \varepsilon_2} - \cancel{\varepsilon_1} + \cancel{\varepsilon_1 \varepsilon_2}}{\cancel{1} - \cancel{1} + \varepsilon_2 + \varepsilon_1 - \varepsilon_1 \varepsilon_2} \right]$$

$$= E_1 \left[\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \right]$$

Similarly $\varphi_2 = E_2 \left[\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \right]$

Net heat exchange betn two surface

$$\varphi_{1-2} = \varphi_1 - \varphi_2$$

$$= \frac{E_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} - \frac{E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$E_b = \sigma T^4$

$E = \varepsilon \sigma T^4$

$E_1 = \varepsilon_1 \sigma T_1^4$

$E_2 = \varepsilon_2 \sigma T_2^4$

$$\varphi_{1-2} = \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \sigma T_1^4 \varepsilon_2 - \varepsilon_2 \sigma T_2^4 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 + \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2 (T_1^4 - T_2^4)\sigma}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \cdot \sigma (T_1^4 - T_2^4)$$

$$= \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1} - 1} \cdot \sigma (T_1^4 - T_2^4)$$

$$= \underbrace{f_{12}}_{\text{interchange factor.}} \sigma (T_1^4 - T_2^4)$$