Chapter : 6
Fundamental of Compressible Flow

Applied Thermodynamics

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Introduction

Basic Types of fluid flow

1) uniform & non-uniform

2) Steady & unsteady

3) one - two - three

4) $\boxed{\text{Compressible}}$ flow -

Incompressible flow -

$PV = mRT$

ρ = Density = C

flow, nozzle $\rho \neq C$

Steam flow through turbine

Basic Thermodynamic Relation

$PV = mRT$

$P \rightarrow N/m^2$

$V \rightarrow m^3$

$m \rightarrow kg$

$R \rightarrow air = 287\ J/kg\,K$

$R_U = MR \qquad M = 28.97g$

$T = Temp\ K$

$\forall$ = specific volume

$P\frac{V}{m} = RT$

$P\forall = RT$

$\forall = \frac{1}{\rho}$

$\frac{P}{\rho} = RT$

$\boxed{P = \rho RT}$ — I

$P = \rho RT$

$dp = \rho R dT + \rho\, dR\, T + d\rho\, RT$

ρRT

$$\frac{dp}{\rho RT} = \frac{\rho R dT}{\rho RT} + \frac{\rho T dR}{\rho RT} + \frac{RT d\rho}{\rho RT}$$

$$\frac{dp}{\rho RT} = \frac{dT}{T} + \frac{dR}{R} + \frac{d\rho}{\rho}$$

$P = \rho RT \qquad R = C$

$$\frac{dp}{p} = \frac{dT}{T} + \frac{d\rho}{\rho} + 0$$

$$\boxed{\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0}$$

Basic Thermodynamic Relation

$dQ = mc\,dT$

$C = \frac{dQ}{m\,dT}$ → C_p

C_v

air

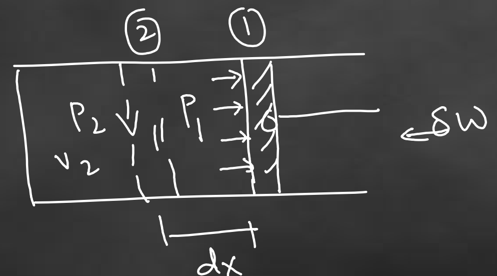
$C_p = 1.005$ kJ/kg K

$C_v = 0.717$ kJ/kg K

γ = Adiabatic index

$= \frac{C_p}{C_v} = \frac{1.005}{0.717} = 1.4$ → air

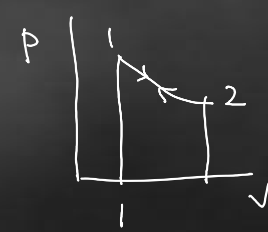
→ mechanical work



δW = Force × Dist

$= P \times A \times dx$

$\delta W = P\,dV$



Basic Thermodynamic Relation

First law

$$\delta Q = \delta W + dE$$

Basic Thermodynamic Process

1) $P = C$ $\qquad \frac{V}{T} = C$

2) $V = C$ $\qquad \frac{P}{T} = C$

3) $T = C$ $\qquad PV = C$

4) $dQ = 0$ $\qquad PV^{\gamma} = C$

5) $\qquad PV^{n} = C$ $\qquad n$ = Polytropic index

Basic Equation for one dimensional Compressible Flow

1) Equation of State → $\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$

$P = \rho R T$

$\frac{P}{\rho} = C$

2) Continuity eq^n

3) Momentum eq^n

4) Energy eq^n

$$\dot{m} = \rho A V = C$$

$$\rho A V = C \quad - \text{I}$$

$$\rho v dA + \rho A dv + A v d\rho = 0$$

$$A = C \rightarrow dA = 0$$

$$\frac{\rho v dA}{\rho v A} + \frac{\rho A dv}{\rho A v} + \frac{A v d\rho}{\rho A v} = 0 \Rightarrow \boxed{\frac{dv}{v} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0}$$

Basic Equation for one dimensional Compressible Flow

momentum flux $= \dot{m} \times v$

$= \rho A v \times v$

$= \rho A v^2$

$\rho A v (v_1 - v_2)$

4) $\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = C \qquad \rho \neq C$

incompressible $\rho = C \qquad \int \frac{dp}{\rho} = \frac{p}{\rho} \Rightarrow$

Compressible $\rho \neq C \qquad \int \frac{dp}{\rho} \neq \frac{p}{\rho}$

Basic Equation for one dimensional Compressible Flow

isothermal process

$$\frac{p}{\rho} = c$$

$$\int \frac{dp}{p/c} + \frac{v^2}{2} + gz = C$$

$$\int \frac{dx}{x} = \ln x$$

$$c\int \frac{dp}{p} + \frac{v^2}{2} + gz = C$$

$$c \ln p + \frac{v^2}{2} + gz = c$$

Basic Equation for one dimensional Compressible Flow

$$\frac{P}{\rho^{\gamma}} = C$$

$$\frac{P}{C} = \rho^{\gamma}$$

$$\rho = (P/C)^{1/\gamma}$$

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = C$$

$$\int \frac{dp}{(P/C)^{1/\gamma}} + \frac{V^2}{2} + gz = C$$

$$C^{1/\gamma} \int P^{-1/\gamma} dp + \frac{V^2}{2} + gz = C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$C^{1/\gamma} \frac{P^{-1/\gamma + 1}}{-\frac{1}{\gamma} + 1} + \frac{V^2}{2} + gz = C \checkmark$$

$$\boxed{\frac{\gamma}{\gamma - 1} \frac{P^{\left(\frac{1}{\gamma} + \frac{\gamma - 1}{\gamma}\right)}}{\rho} + \frac{V^2}{2} + gz = C}$$