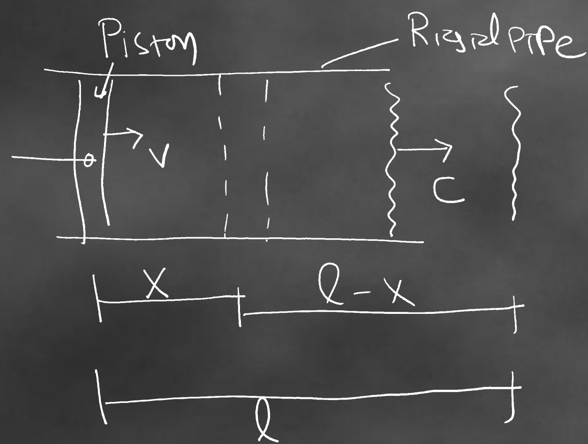
# Pressure wave Propagation and sound Velocity




let $V$ = Velocity of piston

$C$ = Velocity of Pressure wave

$P$ = Pressure before movement of piston

$P + dP$ = fluid Pressure after movement

$\rho$ = Density before compression

$\rho + d\rho$ = Density after Compression

$dt$ = Small interval of time

$x$ = Distance Travel by Piston

$l$ = Distance Travel by Pressure wave

$$V = \frac{x}{dt} \Rightarrow x = v\,dt$$

$$C = \frac{l}{dt} \Rightarrow l = c\,dt$$

# Pressure wave Propagation and sound Velocity

Continuity eqn

mass of fluid before compression = mass of fluid after comp

$$\rho A l = (\rho + d\rho) A (l - x)$$

$$\rho A l = (\rho + d\rho) A (c\,dt - v\,dt)$$

$$\rho \cancel{A} c \cancel{dt} = (\rho + d\rho) \cancel{A} (c - v) \cancel{dt}$$

$$\rho c = (\rho + d\rho)(c - v)$$

$$\cancel{\rho c} = \cancel{\rho c} - \rho v + c\,d\rho - v\,d\rho$$

$$c\,d\rho = \rho v - \underline{v\,d\rho} \qquad d\rho \rightarrow \text{small}$$

$$c\,d\rho = \rho v \quad \text{— (I)}$$

# Pressure wave Propagation and sound Velocity

Rate of change of momentum = force applied

$$\frac{\rho \forall}{dt}(V-0) = (P+dP)A - P \times A$$

$$\frac{\rho \cancel{A} V l}{dt} = dP \times \cancel{A}$$

$$\frac{\rho l V}{dt} = dP \qquad \frac{\rho\, c\, \cancel{dt}\, V}{\cancel{dt}} = dP$$

$$\boxed{\rho V = \frac{dP}{c}} \text{ — II}$$

$$c\, d\rho = \rho V$$

$$\rho V = \frac{dP}{c} \Rightarrow \text{from eqn I \& II} \quad c\, d\rho = \frac{dP}{c} \Rightarrow c^2 = \frac{dP}{d\rho}$$

$$\boxed{c = \sqrt{dP/d\rho}}$$

# Sound Velocity in Terms of Bulk Modulus

$C = \sqrt{dP/d\rho}$

$\rho \forall = C$

$\rho\, d\forall + \forall\, d\rho = 0$

$\frac{d\forall}{\forall} + \frac{d\rho}{\rho} = 0$

$-\frac{d\forall}{\forall} = \frac{d\rho}{\rho} \quad \text{— II}$

Bulk modulus K

$K = \frac{\text{Change in Pressure}}{\text{Volumetric strain}} = -\frac{dp}{\left(\frac{dV}{V}\right)}$

$K = -\frac{dp}{\frac{d\forall}{\forall}} \Rightarrow \frac{d\forall}{\forall} = -\frac{dp}{K} \quad \text{— I}$

$B = \text{Compressibility} = \frac{1}{K}$

$\frac{dp}{K} = \frac{d\rho}{\rho} \Rightarrow \frac{dp}{d\rho} = K/\rho \Rightarrow \boxed{C = \sqrt{K/\rho}}$

# Velocity of Sound for Isothermal Process

$C = \sqrt{dP/d\rho}$

Isothermal process $\frac{P}{\rho} = C$ $\quad P\rho^{-1} = C$

$$P(-1)\rho^{-2}d\rho + \rho^{-1}dP = 0$$

$$-\frac{P}{\rho^2}d\rho + \frac{1}{\rho}dP = 0$$

$$-Pd\rho + \rho dP = 0$$

$$Pd\rho = \rho dP$$

$$\frac{P}{\rho} = \frac{dP}{d\rho}$$

$C = \sqrt{P/\rho}$

$K = P \rightarrow$ Isothermal Process

# Velocity of Sound for Adiabatic Process

$c = \sqrt{dP/d\rho}$

Adiabatic Process $P/\rho^{\gamma} = C$ $P\rho^{-\gamma} = C$

$$P(-\gamma)\rho^{-\gamma+1} d\rho + \rho^{-\gamma} dP = 0$$

$$-\frac{\gamma P \, d\rho}{\rho^{\gamma}\rho^{-1}} + \frac{1}{\rho^{\gamma}} dP = 0$$

$$-\gamma P \, d\rho + \rho \, dP = 0$$

$$\gamma P \, d\rho = \rho \, dP$$

$$\frac{dP}{d\rho} = \frac{\gamma P}{\rho}$$

$\frac{P}{\rho} = RT$

$$\frac{dP}{d\rho} = \gamma RT$$

$$\Rightarrow \boxed{c = \sqrt{\gamma RT}}$$

# Mach Number and Flow Regimes

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{K A}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

$$C = \sqrt{K/\rho}$$

$$M = \frac{V}{C} = \frac{\text{Velocity of fluid OR object moving in fluid}}{\text{Velocity of sound}}$$

(1) $M < 0.3$ $V <<< C$ incompressible flow

(2) $0.3 < M < 0.9$ $V \leq C$ Subsonic flow

(3) $0.9 < M < 1.1$ Transonic flow

(4) $M = 1 \Rightarrow V = C$ Sonic flow

(5) $M > 1 \Rightarrow V > C$ Supersonic flow

(6) $M > 7 \Rightarrow V >> C$ Hypersonic flow