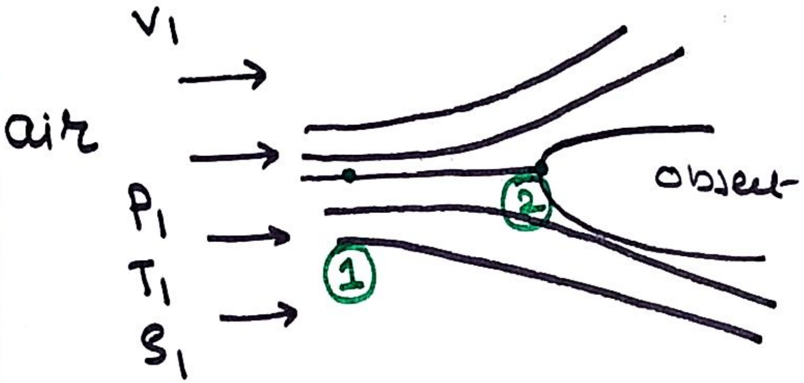
→ Stagnation Properties



Velocity becomes 0 isentropically

K.E → P.E (Adiabatic Process)

Applying Bernoulli's eqn 1 & 2

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$z_1 = z_2$

Point 2 is stagnation point

$P_2 = P_0 \quad \rho_2 = \rho_0 \quad V_2 = 0$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1 \cancel{g}} + \frac{V_1^2}{2\cancel{g}} = \frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0 \cancel{g}} + 0$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1} + \frac{V_1^2}{2} = \frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0}$$

$$\frac{\gamma}{\gamma-1}\left[\frac{P_1}{\rho_1} - \frac{P_0}{\rho_0}\right] = -\frac{V_1^2}{2}$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1 - \frac{P_0}{\rho_0}\cdot\frac{\rho_1}{P_1}\right] = -\frac{V_1^2}{2}$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1 - \frac{P_0}{P_1}\cdot\frac{\rho_1}{\rho_0}\right] = -\frac{V_1^2}{2}$$

→ Adiabatic Process

$$\frac{P}{\rho^\gamma} = c$$

$$\frac{P_1}{\rho_1^\gamma} = \frac{P_0}{\rho_0^\gamma} \rightarrow \frac{\rho_1}{\rho_0} = \left(\frac{P_1}{P_0}\right)^{1/\gamma}$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1 - \frac{P_0}{P_1}\cdot\left(\frac{P_1}{P_0}\right)^{1/\gamma}\right] = -V_1^2/2$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1 - \left(\frac{P_0}{P_1}\right)^{1-1/\gamma}\right] = -V_1^2/2$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1-(P_0/P_1)^{\frac{\gamma-1}{\gamma}}\right] = -V_1^2/2$$

$$1-(P_0/P_1)^{\gamma-1/\gamma} = -\frac{V_1^2}{2}\times\frac{\gamma-1}{\gamma}\times\frac{\rho_1}{P_1}$$

→ Velocity of sound Adiabatic Process

$$C=\sqrt{\gamma RT}$$

Point 1 $\quad C_1=\sqrt{\gamma RT_1}$

$$C_1=\sqrt{\gamma\frac{P_1}{\rho_1}}$$

$$C_1^2=\gamma P_1/\rho_1$$

$$\frac{P_1}{\rho_1}=\frac{C_1^2}{\gamma}$$

$$1-(P_0/P_1)^{\frac{\gamma-1}{\gamma}} = -\frac{V_1^2}{2}\times\frac{\gamma-1}{\cancel{\gamma}}\times\frac{\cancel{\gamma}}{C_1^2}$$

$$\left(\frac{P_0}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1+\frac{V_1^2}{C_1^2}\times\frac{\gamma-1}{2}$$

→ from $M=\frac{V}{C}$

$$M_1=V_1/C_1$$

$$\left(\frac{P_0}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1+M_1^2\times\frac{\gamma-1}{2}$$

$$\boxed{P_0 = P_1\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\gamma/\gamma-1}}$$

→ Stagnation Density

$$\frac{P}{\rho^\gamma}=C$$

$$\frac{P_1}{\rho_1^\gamma}=\frac{P_0}{\rho_0^\gamma}$$

$$\rho_0=\rho_1\left(\frac{P_0}{P_1}\right)^{1/\gamma}$$

$$\rho_0=\rho_1\left[\frac{\cancel{P_1}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\cancel{\gamma}/(\gamma-1)}}{\cancel{P_1}}\right]^{1/\cancel{\gamma}}$$

$$\boxed{\rho_0=\rho_1\left[1+\frac{\gamma-1}{2}M_1^2\right]^{1/\gamma-1}}$$

→ Stagnation Temp

$$\frac{P}{\rho} = RT$$

$$\frac{P_0}{\rho_0} = RT_0$$

$$T_0 = \frac{P_0}{\rho_0 R}$$

$$= \frac{1}{R} \frac{P_0}{\rho_0}$$

$$T_0 = \frac{1}{R} \frac{P_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/\gamma-1}}{\rho_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{1/\gamma-1}}$$

$$= \frac{P_1}{R\rho_1} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/\gamma-1 - \frac{1}{\gamma-1}}$$

$$\boxed{T_0 = T_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)}$$

→ enthalpy at stagnation point

$$h_0 = C_p T_0$$

$$\boxed{h_0 = C_p T_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)}$$

→ velocity of sound at stagnation

$$C_1 = \sqrt{\gamma R T_1}$$

$$C_0 = \sqrt{\gamma R T_0}$$

$$C_0 = C_1 \sqrt{\frac{RT_1}{RT_0}}$$

$$= C_1 \left(\frac{T_1}{T_0}\right)^{1/2}$$

$$\boxed{C_0 = C_1 \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{1/2}}$$