→ Area velocity Relation for Compressible flow

* Continuity eqn.

$$\rho A V = C$$

$$\rho A dV + \rho V dA + V A d\rho = 0$$

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$$

* Euler's eqn for Compressible flow

$$\frac{dP}{\rho} + V dV + g dz = 0$$

$$dz = 0$$

$$\frac{dP}{\rho} + V dV = 0$$

Sonic velocity $c = \sqrt{dP/d\rho}$

$$c^2 = dP/d\rho$$

$$dP = c^2 d\rho$$

$$\frac{c^2 d\rho}{\rho} + V dV = 0$$

$$\frac{d\rho}{\rho} = -\frac{V}{c^2} dV$$

$$\frac{dV}{V} + \frac{dA}{A} - \frac{V}{c^2} dV = 0$$

$$\frac{dA}{A} = \frac{V}{c^2} dV - \frac{dV}{V}$$

$$= \frac{dV}{V}\left[\frac{V^2}{c^2} - 1\right]$$

$M = V/c \quad M^2 = V^2/c^2$

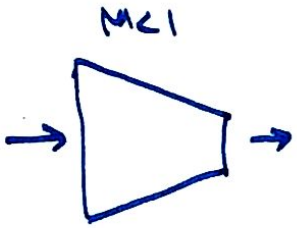
$$\boxed{\frac{dA}{A} = \frac{dV}{V}\left[M^2 - 1\right]}$$

Case -I $M < 1$ Subsonic

$M^2 - 1 < 0$

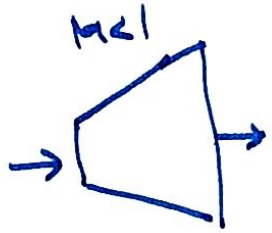
(i) $\frac{dv}{v} > 0$ $\frac{dA}{A} < 0$

(ii) $\frac{dv}{v} < 0$ $\frac{dA}{A} > 0$



$\frac{dA}{A} < 0$

$\frac{dv}{v} > 0$



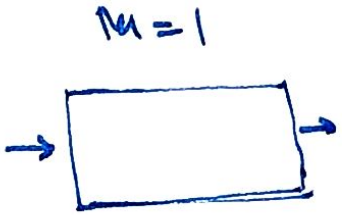
$\frac{dA}{A} > 0$

$\frac{dv}{v} < 0$

Case-II $M = 1$

$M^2 - 1 = 0$

$\frac{dA}{A} = 0$



$\frac{dA}{A} = 0$

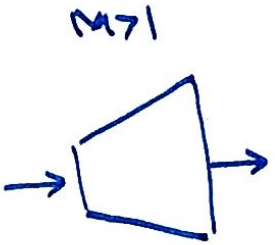
$\frac{dv}{v} = 0$

Case-III $M > 1$

$M^2 - 1 > 0$

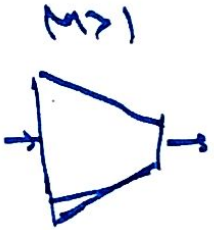
(i) $\frac{dv}{v} > 0$ $\frac{dA}{A} > 0$

(ii) $\frac{dv}{v} < 0$ $\frac{dA}{A} < 0$



$\frac{dv}{v} > 0$

$\frac{dA}{A} > 0$



$\frac{dA}{A} < 0$

$\frac{dv}{v} < 0$