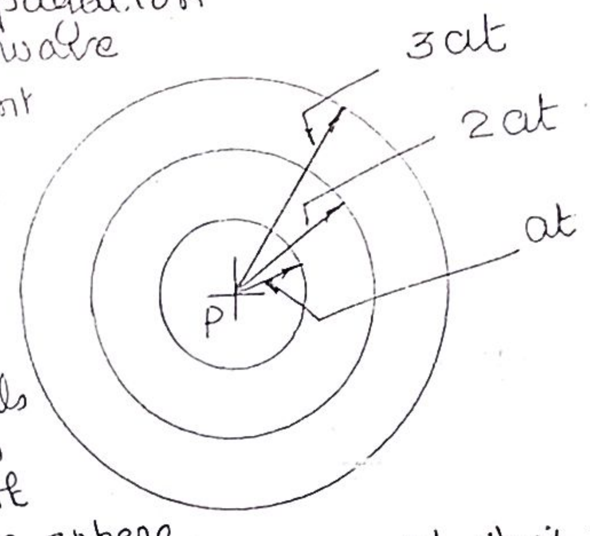
PRESSURE DISTURBANCES IN COMPRESSIBLE FLOW

The basic difference in the phenomenon associated with subsonic and supersonic speeds can be demonstrated qualitatively as follows.

(i) Propagation of sound waves from stationary point source, i.e $C = 0$

Consider the propagation of a spherical sound wave from a stationary point source P. Assume that the fluid surrounding the point is at rest. The pressure pulses which are emitted from the source at regular intervals spread uniformly with the velocity of sound. At time 't' the radii of the sphere is 'at', at time 2t, it is 2at, at time 3t it is 3at and so on.

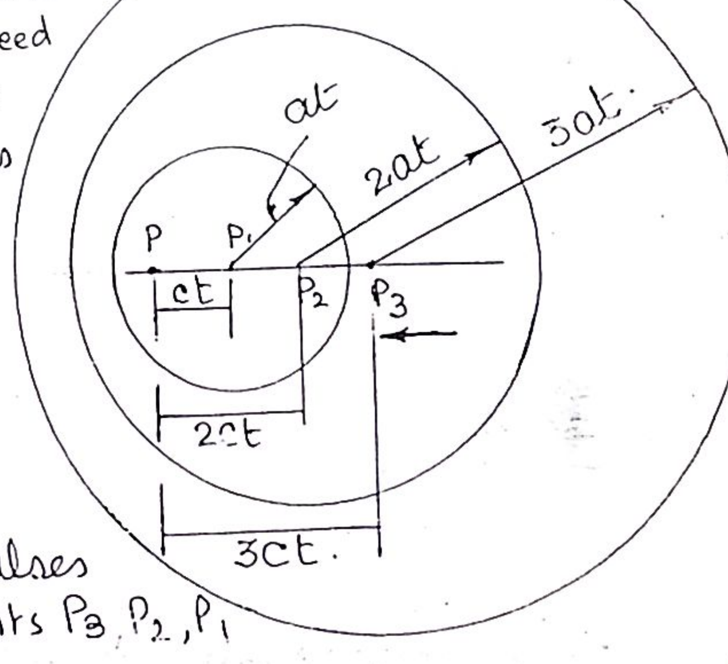


If stones are thrown at the same point at regular intervals, similar pattern will be observed.

(ii) Pressure disturbances produced by a body moving with subsonic speed i.e $C < a$.

Now consider the case when the body moves with uniform subsonic speed of C and assume that the pressure disturbances by the moving body are small so that they may be treated as sound waves.

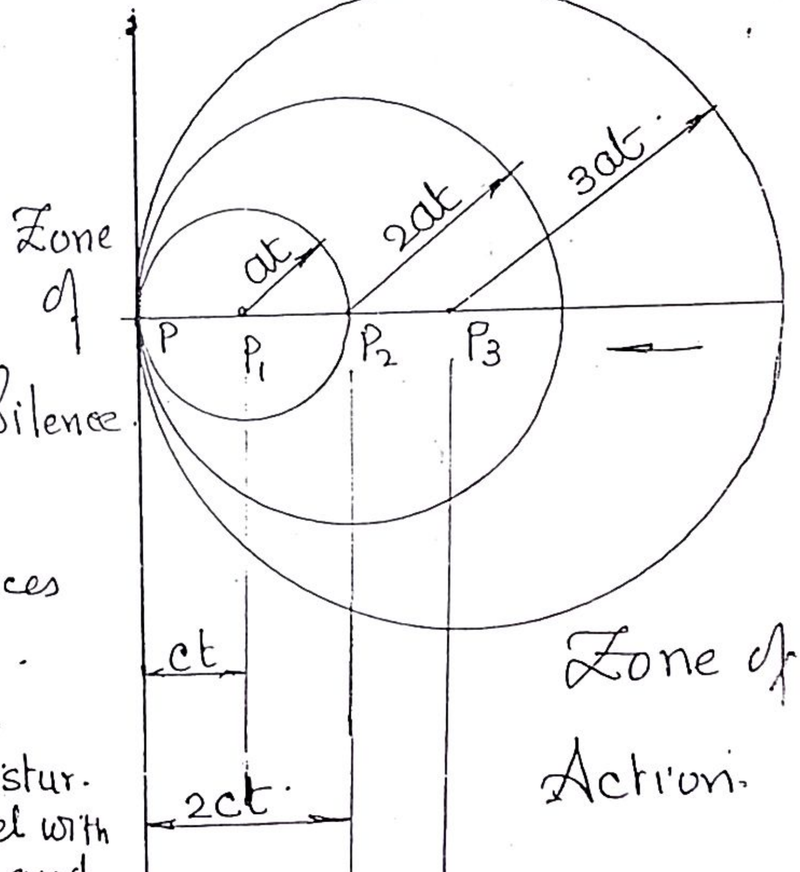
Let P_3 be the point from which the object moves to left with velocity C. Let the pulses be emitted from points P_3, P_2, P_1 as shown



In time t, the object would move to position P_2, in time t_2, to position P_1 and in time 3t to position P as shown in figure. The distances PP_1, PP_2, PP_3 being ct, 2ct, 3ct respectively. The corresponding disturbances created would spread uniformly as shown. At point P_3, the disturbance would have a radius of 3at in time t_3, at P_2 it would have a radius of 2at in time t_2 and at P_1 it would have a radius of at in time t_1. ($t_3 = 3t$, $t_2 = 2t$; $t_1 = t$).

Thus the pressure waves are felt in all the directions but the intensities are not uniform as in previous case.

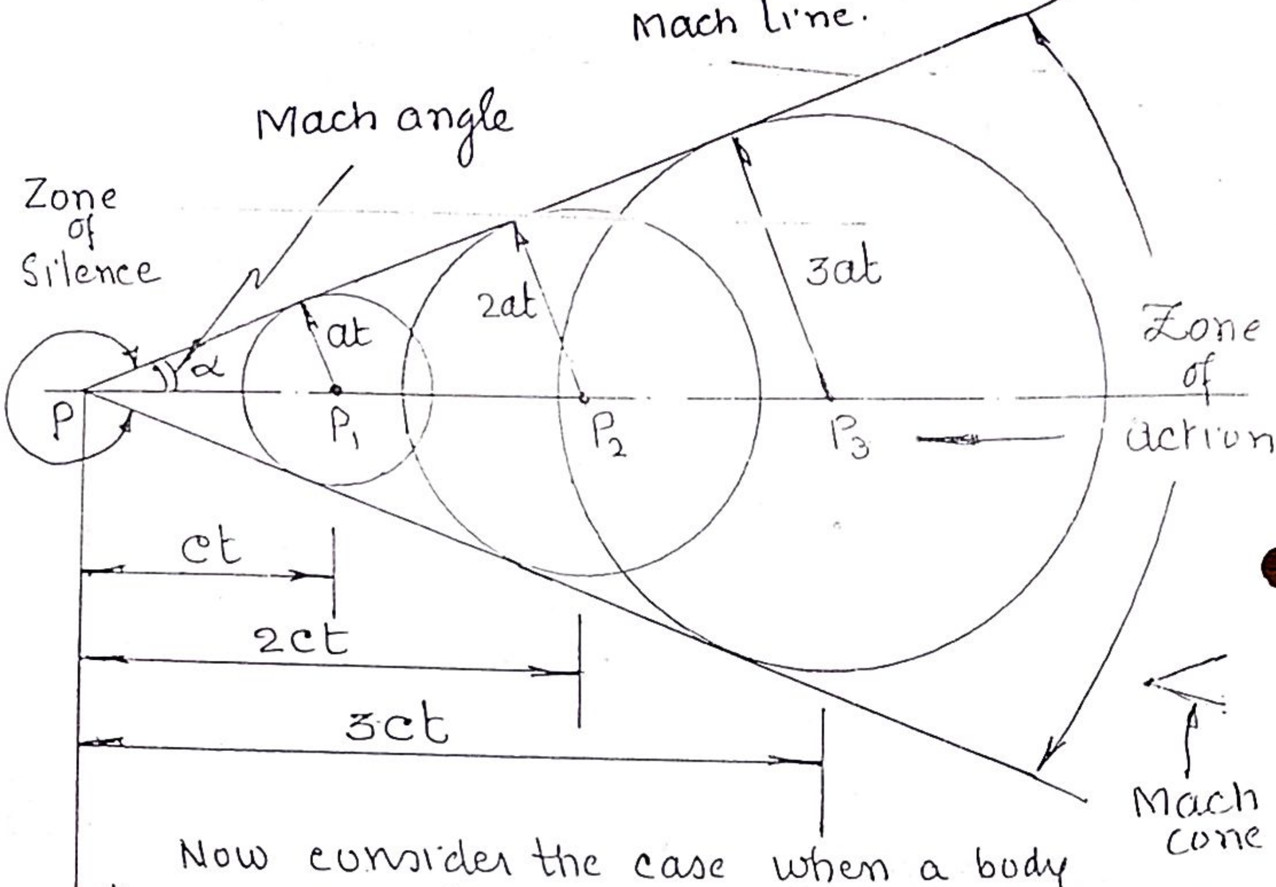
(iii) Pressure wave produced by a body moving at sonic speed i.e $c = a$.



Consider now the case when the object moves with the velocity of sound. Start from point P_3 and move to the left. It would reach pt P in 3t time; P_1 in 2t time and P_2 in time t, the distances PP_1, PP_2, PP_3 being ct, 2ct and 3ct. At the same time distur-bances would travel with the speed of sound and would have radii at, 2at and 3at at shown.

Since $c = a$, the disturbances created by the body are contained in half the plane to the right of body. For this reason, the zone of Silence & zone of action are created as shown. In zone of Silence the pi... disturbances cannot be heard.

Pressure disturbances produced by a body moving with Supersonic Speed. $c > a$.



Now consider the case when a body is moving with supersonic speed i.e $c > a$. Since $c > a$, the disturbance created by the body lags behind the point of body that created the disturbance and as such the disturbance wave front cannot overtake the cone called **Mach cone** whose apex is P. The generator of the cone is called Mach line and the angle α, as shown, is called Mach angle.

$$\text{Now } \sin\alpha = \frac{at}{ct} = \frac{2at}{2ct} = \frac{3at}{3ct} = \frac{1}{M}$$

$$\text{or } \alpha = \sin^{-1}\frac{1}{M}.$$

The Zone of Silence and Zone of action are as shown in figure.

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(2.)

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Steady, one-dimensional, Isentropic flow of perfect gas.

Continuity Eqn.: $\rho AC = \text{const.}$

taking log. on both sides and differentiating we have.

$\log \rho + \log A + \log C = \text{Const}$ or $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dc}{c} = 0$ (a)

Momentum Eqn:

$$\frac{\partial c_x}{\partial t} + c_x \frac{\partial c_x}{\partial x} + c_y \frac{\partial c_x}{\partial y} + c_z \frac{\partial c_x}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$\frac{\partial c_x}{\partial t} = 0$ as flow is steady, $c_y = c_z = 0$ as 1-Dim.

$g_x = 0$ as no body forces.

$$\therefore \quad c_x \frac{\partial c_x}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$c\, dc = -\frac{dP}{\rho} \quad \text{or} \quad \frac{dc}{c} = -\frac{dP}{\rho c^2} \quad \text{— (b)}$$

Isentropic Law gives $P = c' \rho^{\gamma}$ $c' = \text{Const.}$

$$dP = c' \gamma \rho^{\gamma - 1} \times d\rho = c' \rho^{\gamma} \gamma \frac{d\rho}{\rho} = \gamma P \frac{d\rho}{\rho}$$

$$\text{or} \quad \frac{d\rho}{\rho} = \frac{1}{\gamma}\frac{dP}{P} \quad \text{— (c)}$$

Now from eqn (a) & from Eqn (b)

$$\frac{dA}{A} = -\frac{dc}{c} - \frac{1}{\gamma}\frac{dP}{P} = \frac{dP}{\rho c^2} - \frac{1}{\gamma}\frac{dP}{P}$$

$$\frac{dA}{A} = \frac{dP}{\rho c^2}\left(1 - \frac{\rho c^2}{\gamma P}\right) \quad \text{but } \frac{\gamma P}{\rho} = a^2 \ \& \ M^2 = \frac{c}{c}$$

$$\boxed{\frac{dA}{A} = \frac{dP}{\rho c^2}\left(1 - M^2\right)}$$

Also $\frac{dc}{c} = -\frac{dP}{\rho c^2} = -\frac{dP}{d\rho}\frac{d\rho}{\rho}\frac{1}{c^2} = -\frac{a^2}{c^2}\frac{d\rho}{\rho}$

$$\text{or} \quad \frac{d\rho}{\rho} = -M^2 \frac{dc}{c}$$

Again $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dc}{c} = 0$

$$-M^2 \frac{dc}{c} + \frac{dA}{A} + \frac{dc}{c} = 0 \quad \text{or} \quad \frac{dc}{c} = -\frac{dA/A}{(1-M^2)}$$

Thus, we have.

$$\boxed{\frac{dc}{c} = -\frac{dA/A}{(1-M^2)}} \quad \text{and} \quad \boxed{\frac{dA}{A} = \frac{dP}{\rho c^2}(1-M^2)}$$

Analysis:-

(a) When $M > 1$ (Supersonic), $(1-M^2)$ is negative then

(i) If dA is positive (i.e diverging passage), dP is negative and $\frac{dc}{c}$ is positive

∴ it gives accelerating expansive flow.

(ii) If dA is negative (i.e converging passage. dP is positive and $\frac{dc}{c}$ is negative.

∴ it gives retarding compressible flow.

(b) When $M < 1$ (Subsonic), $(1-M^2)$ is positive then

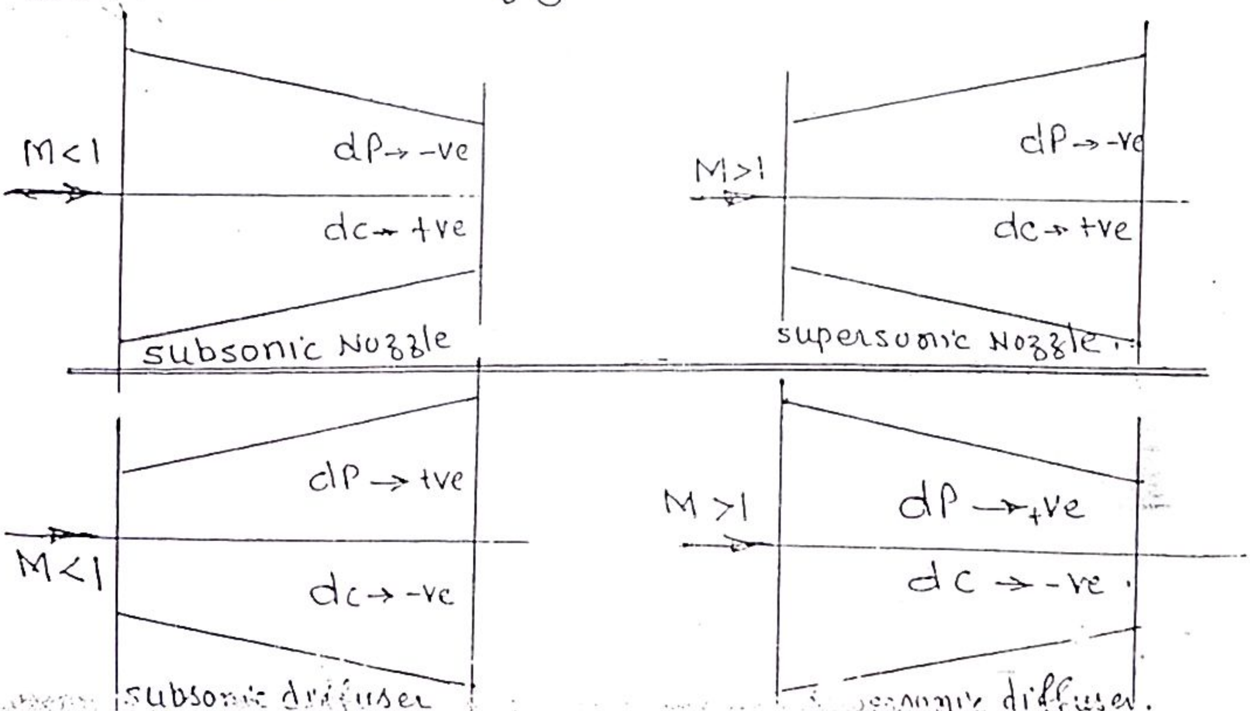
(i) If dA is positive (diverging passage) dP is positive and $\frac{dc}{c}$ is negative.

∴ it gives retarding compressible flow.

(ii) If dA is negative (converging passage) dP is negative and $\frac{dc}{c}$ is positive

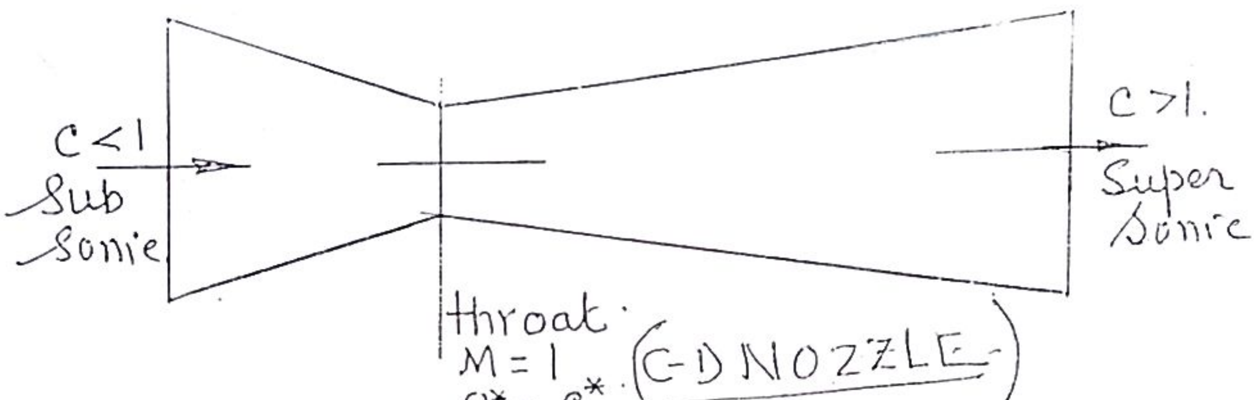
∴ It gives accelerating expansive flow.

These cases of subsonic & supersonic flow are shown in figure below.



(c) When $M = 1$, equations indicates that dA must be zero. Thus, for a duct of variable cross-section, the sonic velocity can occur at a section where area is minimum, irrespective whether the initial velocity in the duct is subsonic or supersonic. However, this does not mean that the velocity at a section with minimum area has to be sonic.

Thus, for the flow to change isentropically from supersonic to subsonic or vice versa, the duct must have a convergent section, a throat and a divergent section.



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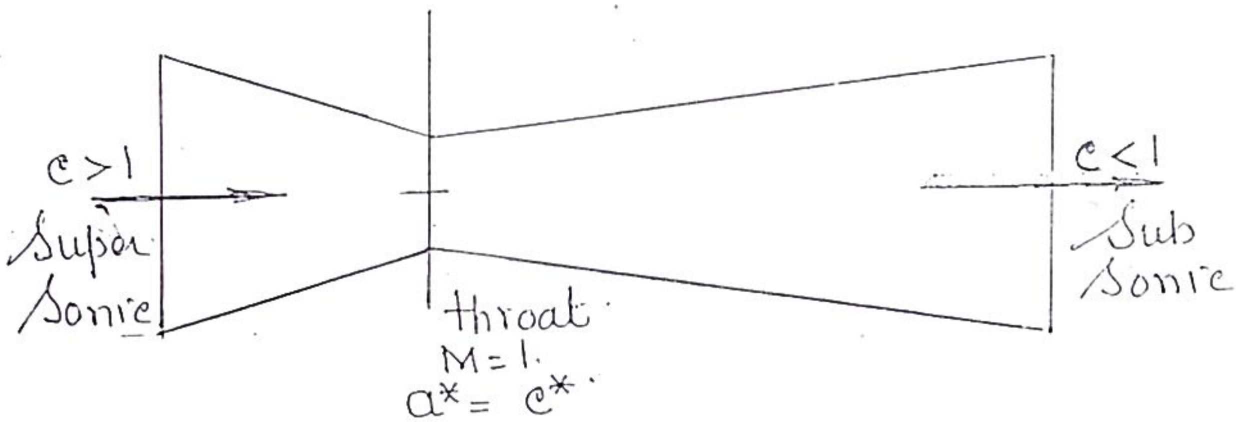

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