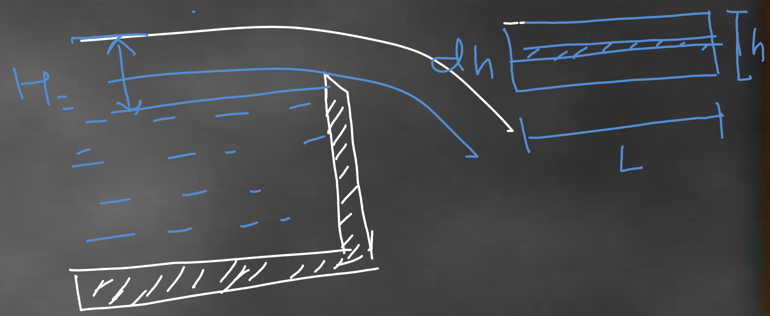
* Discharge over a Rectangle Notch

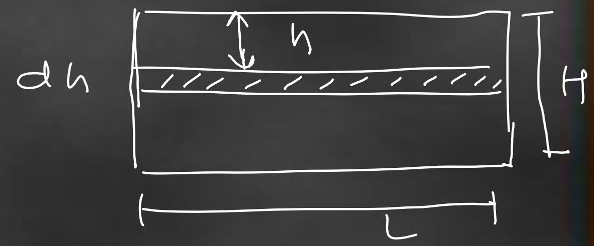


H = head of water

L = length of notch

area of strip = dh × L

Velocity of water at strip (Theoretical) = $\sqrt{2gh}$



Discharge through strip dQ = area × V × Cd

$$dQ = C_d \times dh \times L \times \sqrt{2gh}$$

Total Discharge $Q = \int dQ$

$$= \int_0^H C_d \, dh \times L \times \sqrt{2gh}$$

$$= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} \, dh$$

$$Q = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

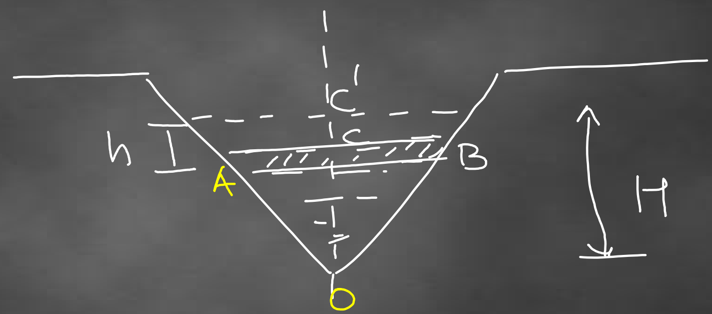
$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g} \left(H^{3/2} - 0 \right)$$

$$\boxed{Q = \frac{2}{3}\, C_d \times L \times \sqrt{2g}\; H^{3/2}}$$

* Discharge over a Triangular Notch

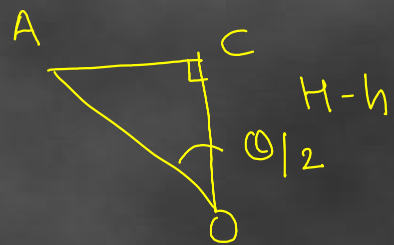
let H = head of water

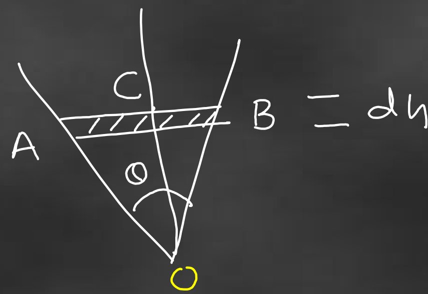
θ = angle of notch



area of strip = $AB \times dh$

$$= 2(H-h)\tan\theta/2 \times dh$$





Velocity of water at strip = $\sqrt{2gh}$

Discharge dQ (strip) = area × Velocity × C_d

$$\tan\theta/2 = \frac{AC}{OC} \Rightarrow AC = OC \times \tan\theta/2$$

$$AC = (H-h) \times \tan\theta/2$$

$$dQ = C_d \times 2(H-h)\tan\theta/2 \times dh \times \sqrt{2gh}$$

$$AB = 2AC = 2 \times (H-h)\tan\theta/2$$

Total Discharge $Q = \int dQ$

$$Q = \int_0^H Cd \times 2(H-h)\tan\theta/2 \, dh \times \sqrt{2gh}$$

$$= 2\,Cd \tan\theta/2 \sqrt{2g} \int_0^H \sqrt{h}\,(H-h)\,dh$$

$$= 2\,Cd \tan\theta/2 \sqrt{2g} \int_0^H (H h^{1/2} - h^{3/2})\,dh$$

$$= 2\,Cd \tan\theta/2 \sqrt{2g} \left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$= 2\,Cd \tan\theta/2 \sqrt{2g} \left[\frac{2 H (H)^{3/2}}{3} - \frac{2 H^{5/2}}{5} \right]$$

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$$Q = 2 C_d \tan\theta_{/2} \sqrt{2g} \left[\frac{2H^{5/2}}{3} - \frac{2H^{5/2}}{5} \right]$$

$$= 2 C_d \tan\theta_{/2} \sqrt{2g} \left[\frac{10-6}{15} H^{5/2} \right]$$

$$\boxed{Q = \frac{8}{15} C_d \tan\theta_{/2} \sqrt{2g} \, H^{5/2}}$$